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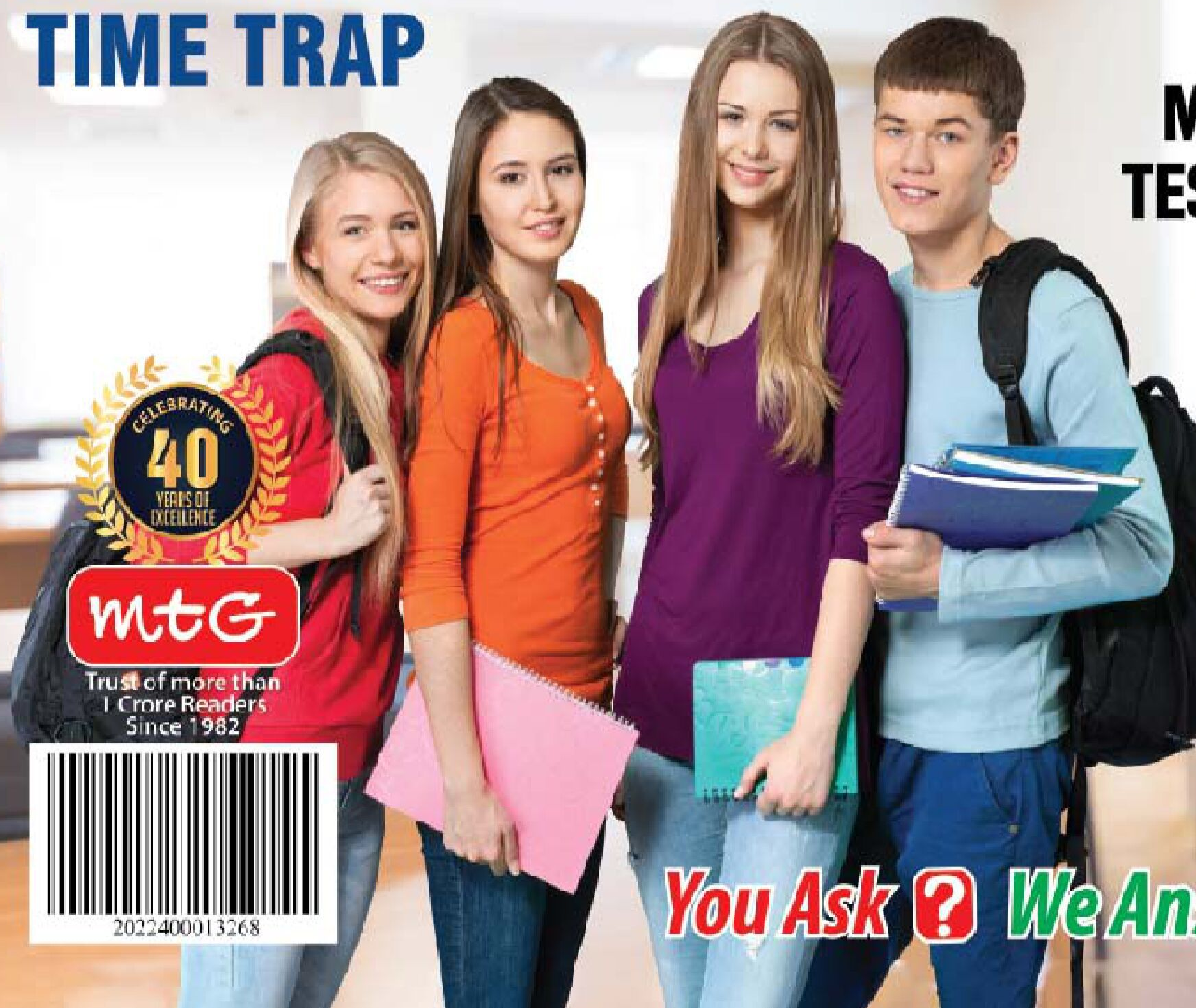

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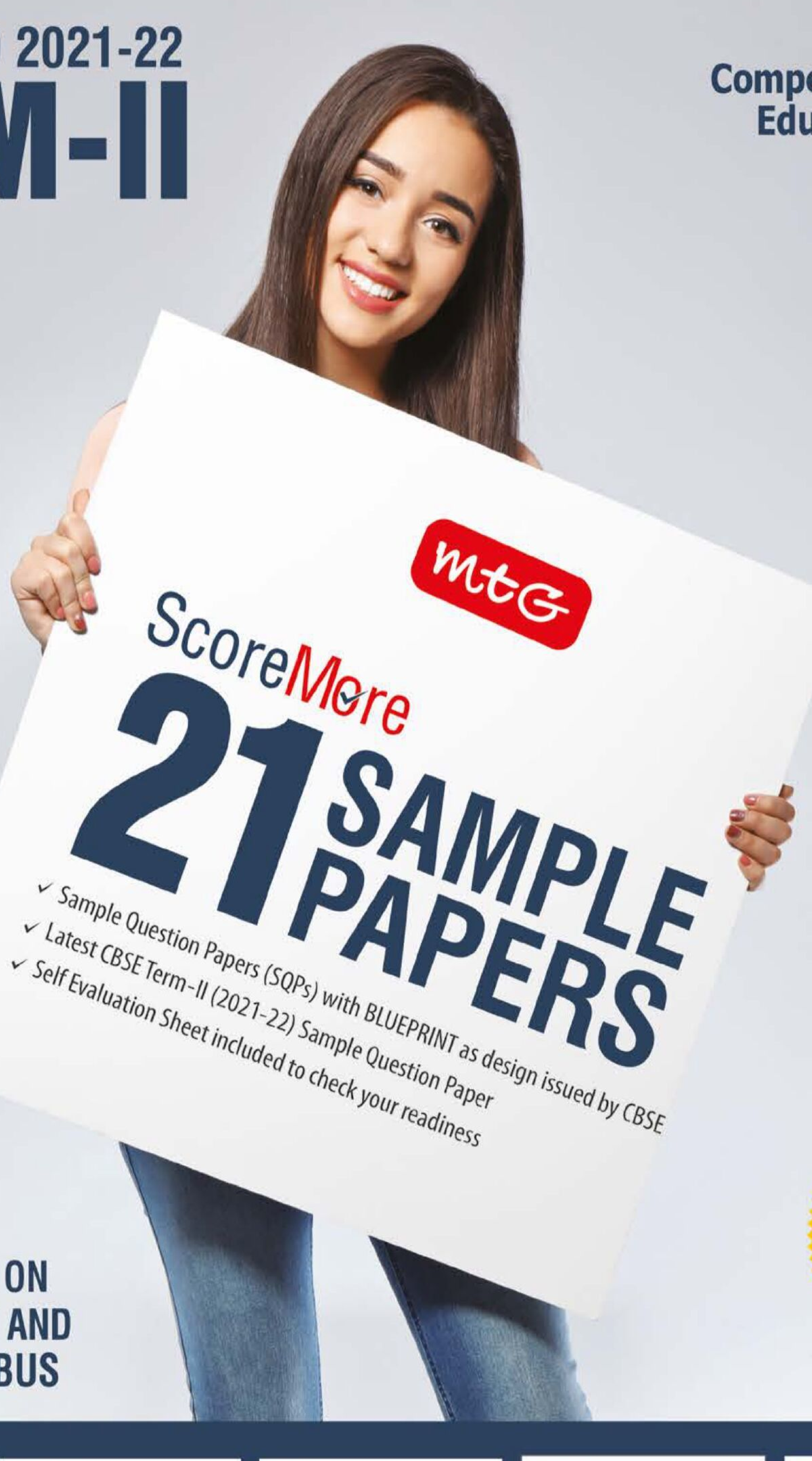
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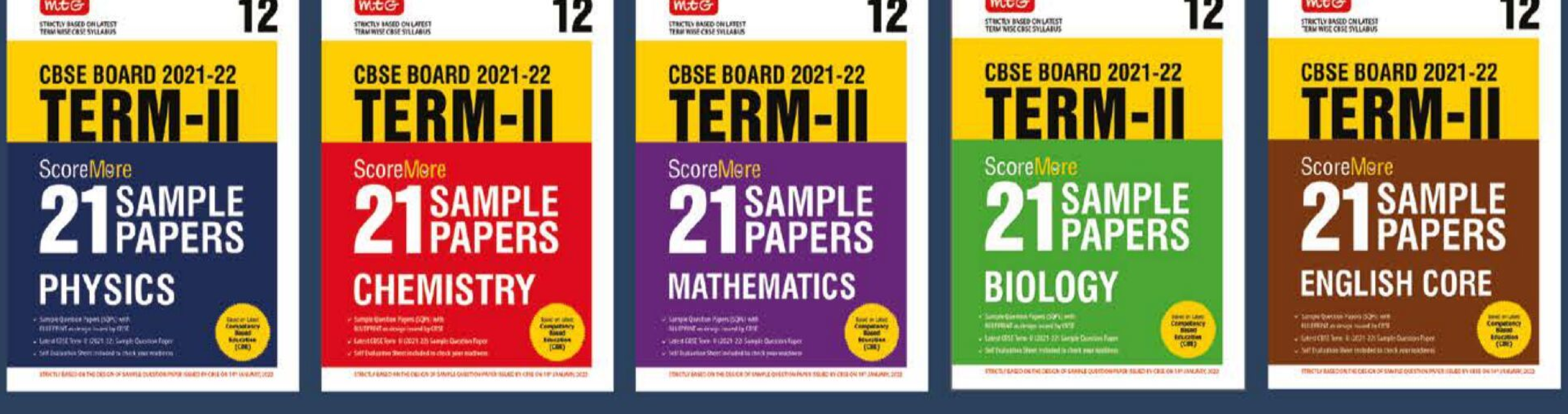
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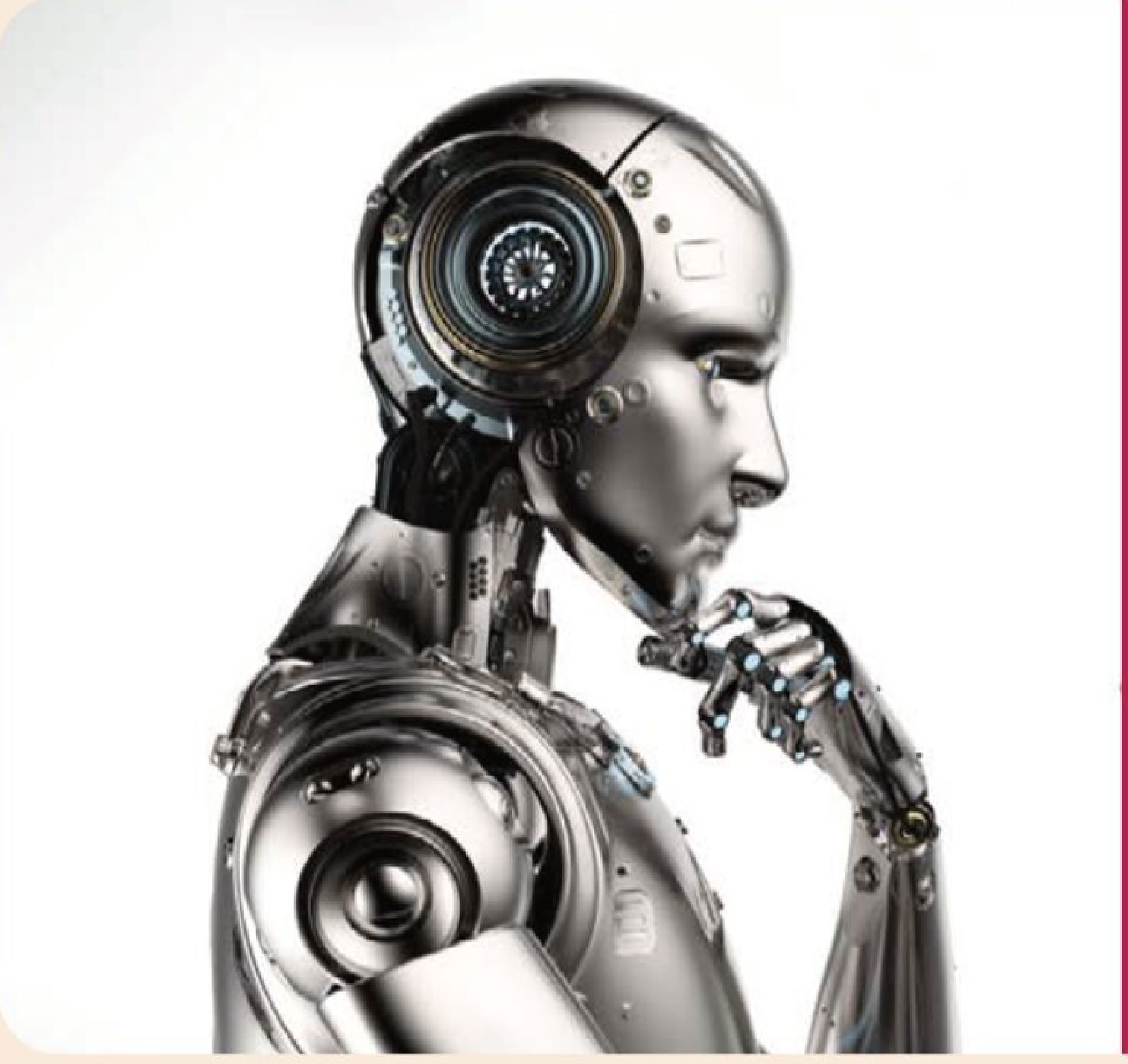

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It’s advantage Mock Tests

Should one take more mock tests or utilise the time revising? This is a question that has often plagued JEE/NEET students’ mind. Find out why taking mock tests is crucial for your preparation and success.

If you ask the students who have cleared JEE or NEET for a mantra, they are most likely to share their belief in the saying ‘practice makes perfect’. A key factor to be JEE/NEET–ready is to take as many mock tests as possible. Don’t let anyone tell you that time spent taking such tests is time away from studying or revising. In fact, it is quite the opposite. Mock tests are the single most important differentiator among students of the same preparation level.

Understand your preparation level

It is important to understand your overall level of preparation. Is your concept base solid? Can you recall the formulas easily? Are you able to timely solve questions based on simple concepts? Are you able to figure out the tricky questions? You need to have answers to questions like these much before the actual exam. Take your time to analyse how well you are prepared and take steps to rectify any weaknesses in your preparation or question solving strategies.

Test your subject–wise expertise

JEE Advanced has subject–wise cut–offs to secure admission in any of the colleges. NEET, on the other hand, has overall cut–off which is high, and you need to score a certain number of marks in each subject. Here, mock tests come to your rescue. Taking them helps you understand your proficiency in each subject. Amazon Academy mock test analytics provide subject–wise percentile so that you can understand how well prepared you are in each subject, apart from your overall preparation. This helps in refocusing on your weaker subjects so that you can work on them better.

Improve speed and accuracy

This point cannot be stressed hard enough. You need to constantly improve your speed and accuracy to perform better in JEE and NEET. Speed is a core element of both. For example, you have to answer 180 questions in 180 minutes in NEET, that is one minute per question on an average. This doesn’t give you a lot of time to think leisurely. You need to solve questions fast, and accurately. There is no scope for silly mistakes. Accuracy is critical for JEE Advanced, which is known for complex and intricate questions. Here too, the more mock tests you have taken, the better prepared you are for solving questions fast under exam pressure.

Revise old topics

The most efficient technique for revision is practice. Mock tests are the perfect platform for you to revise what you have learnt. When you solve questions of a topic that you studied a few days or months back, all your learning and retention is brought to test. You can clearly understand how much you know and what you need to brush up. The clarity of your concepts is also tested. Plus, regularly solving questions of previous topics keeps them fresh and top of mind. It reduces the chances of error in the real exam significantly.

Most full–term courses of JEE and NEET come with test series. If you haven’t enrolled in one, you can also sign up for Test Series separately. Amazon Academy has launched an excellent Mock Test Series for both JEE and NEET that you can enrol in to boost your exam preparation. They also give you access to the Live Test Analysis sessions where you can learn about the optimum question solving tricks from expert Amazon Academy faculty. Irrespective of whatever you choose, keep up the practice.

Disclaimer: The information provided in this article is of a general nature and should not be considered a substitute for professional advice.

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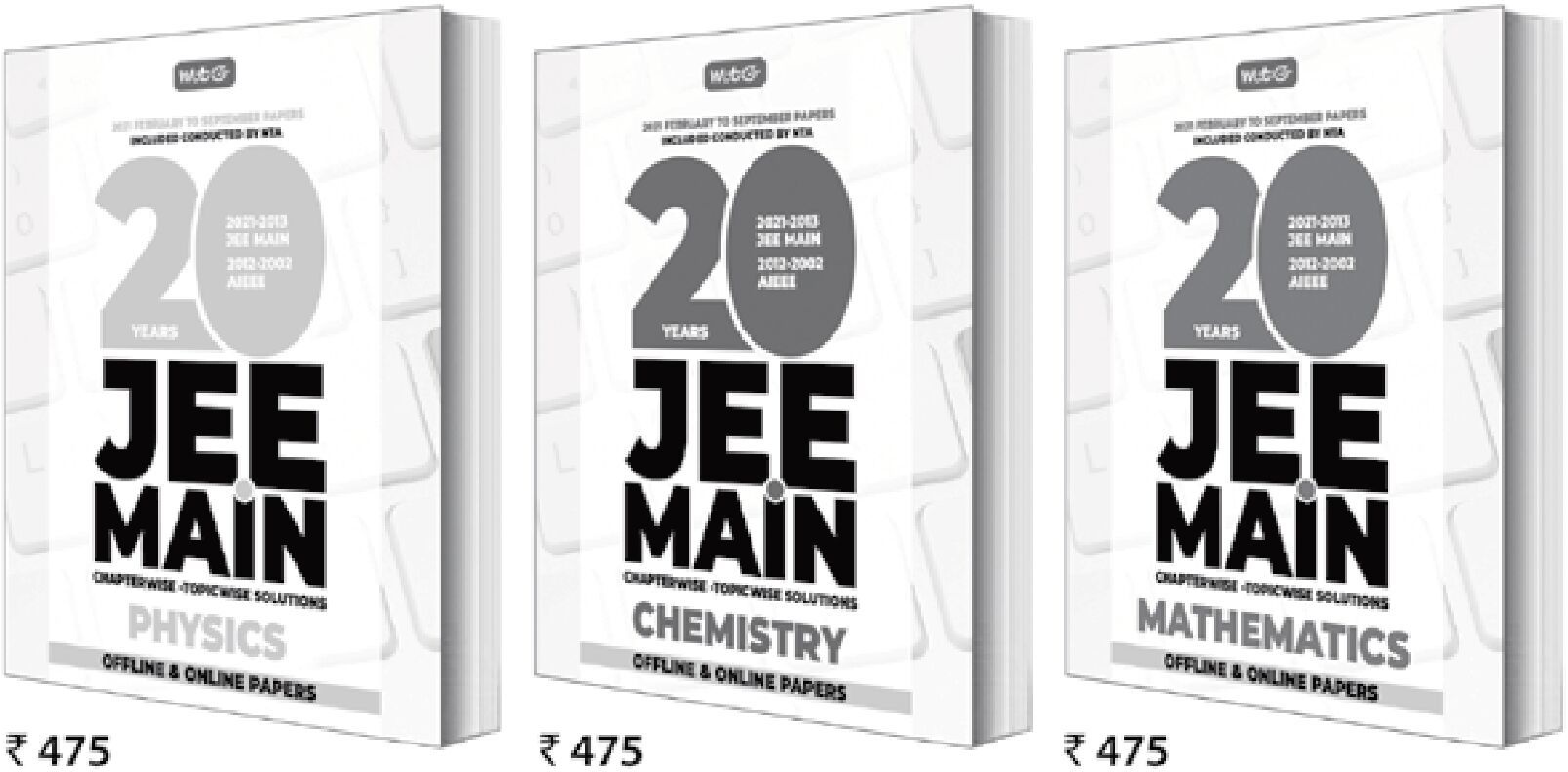
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Editor : Anil Ahlawat

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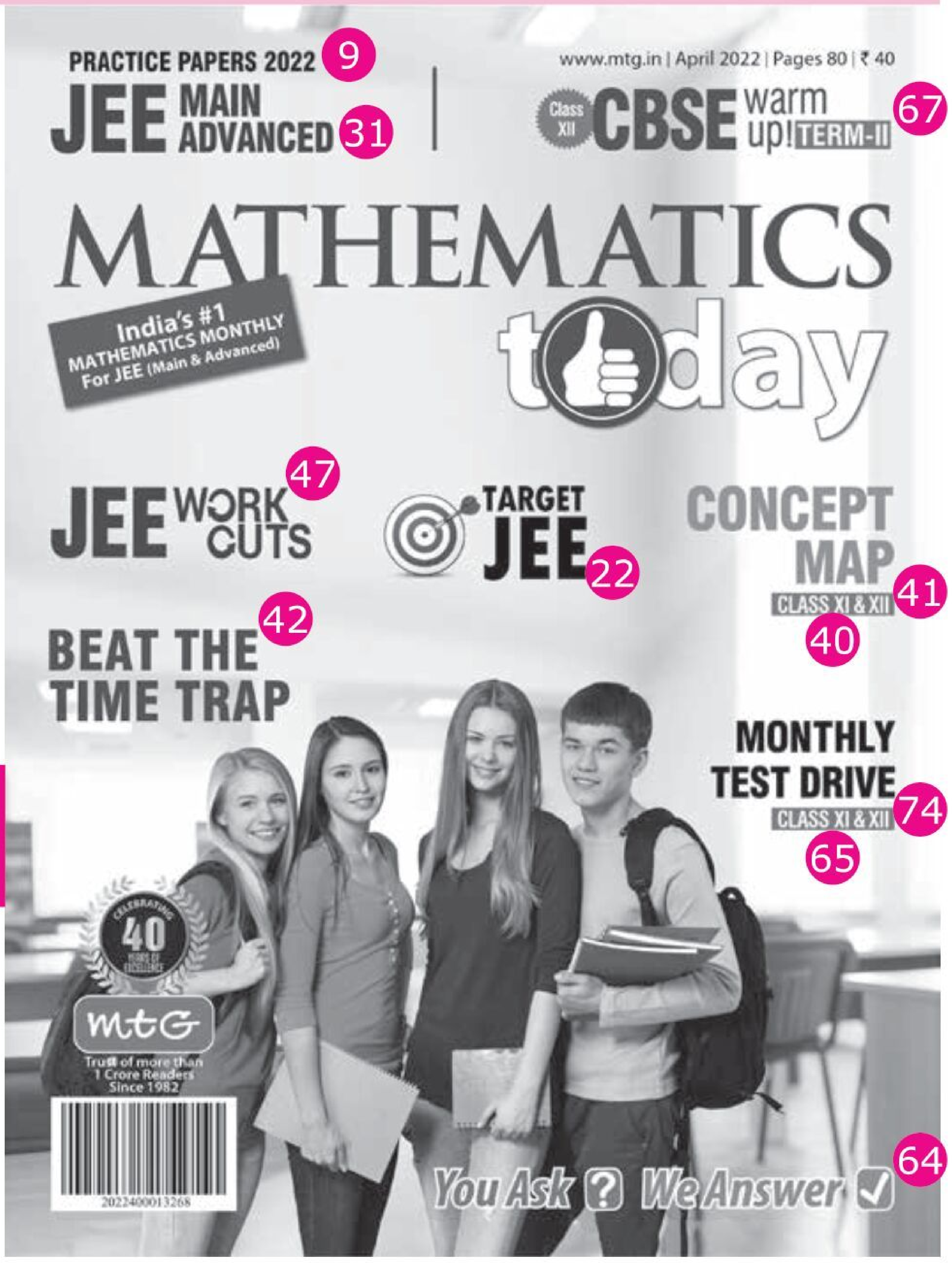
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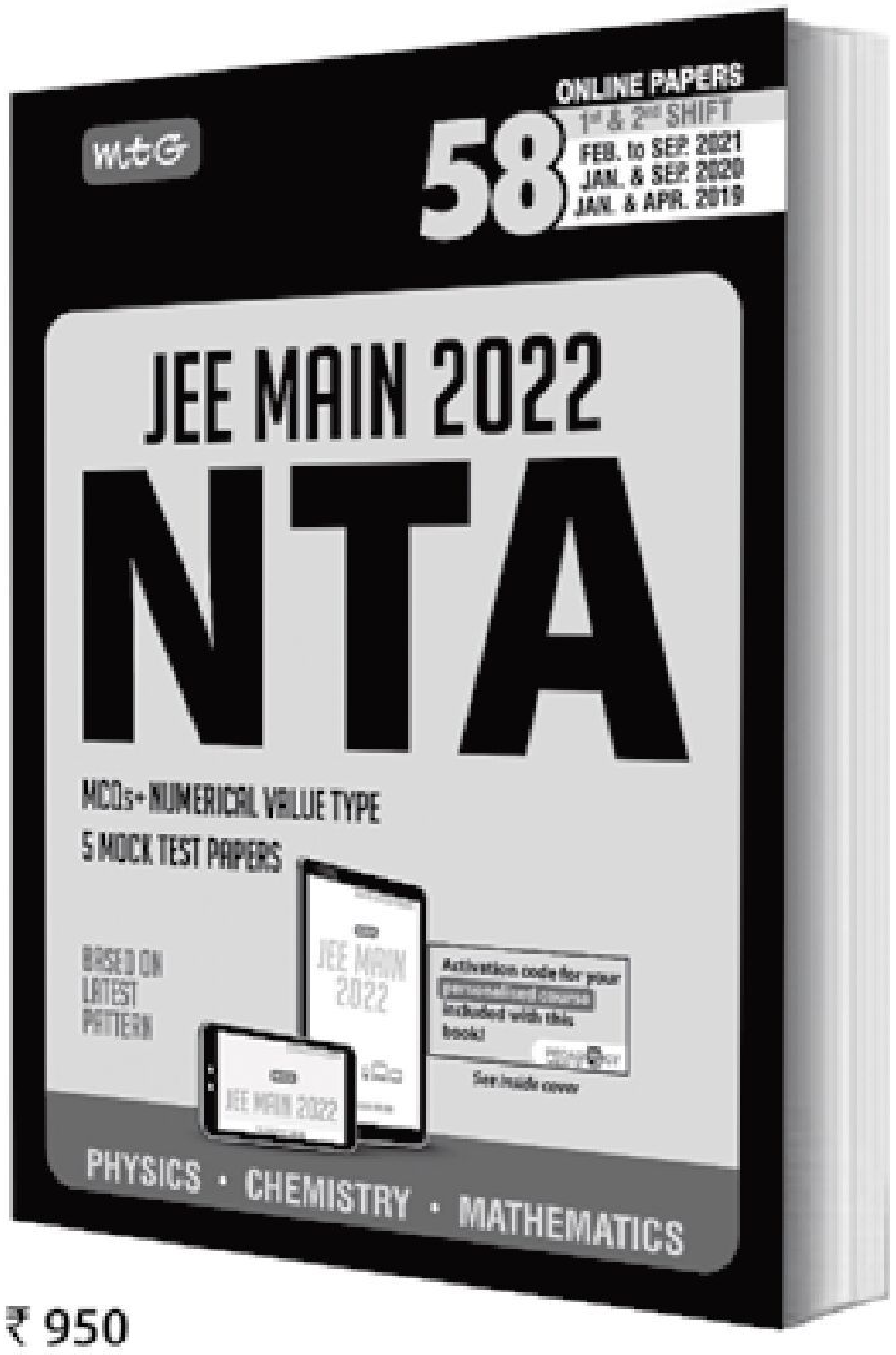

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- 4915 MCQs for Practice
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Exam Dates
Session-1
21, 24, 25, 29 April and 1, 4 May
Session-2
24, 25, 26, 27, 28 and 29 May 2022

SECTION - A

MULTIPLE CHOICE QUESTIONS

1. Let $f(x) = x^2 - bx + c$, b is an odd positive integer, $f(x) = 0$ has two prime numbers as roots and $b + c = 35$. Then the global minimum value of the function $f(x)$ is

(a) $-\frac{183}{4}$ (b) $\frac{173}{16}$ (c) $-\frac{81}{4}$ (d) $\frac{81}{4}$

2. The coefficient of x^6 in the expansion of $\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}\right)^2$ is given by

(a) $\frac{2}{15}$ (b) $\frac{4}{15}$ (c) $\frac{31}{360}$ (d) $\frac{2}{45}$

3. If $f: R \to R$ is defined by $f(x) = x - [x] - \frac{1}{2}\ \forall\, x \in R$, where $[x]$ denotes the greatest integer function, then solution set of $f(x) = 1/2$ is given by

(a) Z, the set of all integers
(b) N, the set of all natural numbers
(c) ϕ, the empty set
(d) R, the set of all real numbers

4. Consider a triangle with sides 3 cm, 6 cm, 8 cm respectively. Now if a man runs around the triangle in such a way that he is always at a distance of 1 cm from the side of triangle, then distance travelled by him is given by

(a) 17 (b) 20 (c) $17 + 2\pi$ (d) $17 + 3\pi$

5. The angle between a pair of tangents drawn to the curve $7x^2 - 12y^2 = 84$ from $M(1, 2)$ is given by

(a) $\pi/4$ (b) $\pi/2$ (c) $\pi/6$ (d) $\pi/3$

6. The position vector of a point in which a line through the origin perpendicular to the plane $2x - y - z = 4$ meet the plane $\vec{r}\cdot(3\hat{i} - 5\hat{j} + 2\hat{k}) = 6$, is

(a) $(1, -1, -1)$ (b) $(-1, -1, 2)$
(c) $(4, 2, 2)$ (d) $\left(\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}\right)$

7. Let S be an interval in which $f(x) = \int_0^x \frac{dt}{1+t^3}$ is an invertible function. Suppose g is the inverse of f, then which of the following statement is true?

(a) $g'(x) = \frac{1}{f(f'(x))}$ (b) $g'(x) = \frac{1}{g(g'(x))}$
(c) $g'(x) = \frac{1}{g(f'(x))}$ (d) $g'(x) = \frac{1}{f'(g(x))}$

8. Let a relation R in the set N of natural numbers be defined as $(x, y) \in R$ if and only if $x^2 - 4xy + 3y^2 = 0$ for all $x, y \in N$. Then the relation R is

(a) Reflexive (b) Symmetric
(c) Transitive (d) Equivalence

9. If $\int_0^{n\pi+t} |\sin x|\,dx = k - \cos t\ (0 \le t \le \pi)$, then the value of k is

(a) $2n$ (b) $4n + 1$ (c) $3n + 1$ (d) $2n + 1$

10. If $\alpha = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then $B = A + A^2 + A^3 + A^4$ is

(a) Singular (b) Non-singular
(c) Symmetric (d) $|B| = 1$

11. Two man A and B fire at a target. Suppose $P(A) = 1/3$ and $P(B) = 1/5$ are the probabilities of hitting the target by A and B respectively. The probability that the target got hit, is

(a) 1/15 (b) 6/15 (c) 7/15 (d) 8/15

12. Two points A and B are taken on the curve $y = \ln x$ whose abscissa are 1 and e respectively. The chord AB is formed. A point on the curve at which the tangent line is parallel to the chord AB is

(a) $(e + 1, \ln(e + 1))$ (b) $(2, \ln 2)$
(c) $(e - 2, \ln(e - 2))$ (d) $(e - 1, \ln(e - 1))$

13. The cubic equation $4x^3 - 33x^2 + 72x + 6 = 0$ has

(a) one positive real root and two complex roots
(b) one negative real root and two complex roots
(c) all three real roots, one negative and two positive
(d) all three real roots, one positive and two negative

14. Let $f(x) = x^4 e^{-x^2}$, $\forall x \in R$. Then $f(x)$ has

(a) Maxima at $x = 0$
(b) Minima at $x = \pm 2$
(c) Minima at $x = \pm 1$
(d) Maxima at $x = \pm\sqrt{2}$

15. Consider a quadratic equation given by $\sum_{r=1}^{n}\left(3x-\frac{1}{2r+1}\right)\left(2x-\frac{1}{2r+5}\right)=0$. The product of roots of the equation is

(a) $\frac{n+1}{6n(2n+5)}$
(b) $\frac{n+1}{36(2n+5)}$
(c) $\frac{n+1}{36n(2n+5)}$
(d) $\frac{n+1}{72n(2n+5)}$

16. Let $f : R \to (-1, 1)$ defined by $f(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}$. Then f is

(a) one-one onto
(b) one-one but not onto
(c) onto but not one-one
(d) Neither one-one nor onto

17. Two sides of a triangle are $2x - y = 0$ and $x + y = 3$. If its centroid is $(2, 3)$, its third side is

(a) $x + 5y - 9 = 0$
(b) $5x - y - 9 = 0$
(c) $x + 5y + 9 = 0$
(d) $5x - y + 9 = 0$

18. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are real positive numbers, $abc = 1$ and $A^TA = I$, then the value of $a^3 + b^3 + c^3$ is

(a) 4
(b) 3
(c) 2
(d) 1

19. If $f(x) = \frac{x}{\sin x}$ and $g(x)=\frac{x}{\tan x}$, where $0 < x \le 1$, then

(a) both $f(x)$ and $g(x)$ are increasing functions
(b) both $f(x)$ and $g(x)$ are decreasing functions
(c) $f(x)$ is increasing
(d) $g(x)$ is increasing

20. The area bounded by $y = 2 - x^2$ and $y^3 = x^2$ is

(a) $\frac{30}{17}$
(b) $\frac{16}{15}$
(c) $\frac{32}{15}$
(d) $\frac{16}{14}$

SECTION - B

NUMERICAL TYPE

Attempt any 5 questions out of 10.

21. If $\alpha \ne 1$, $\alpha^6 = 1$ and $\sum_{r=1}^{6} {}^6C_r . \alpha^{r-1} = x$, then the value of $|x| =$

22. Maximum value of $\log_5(3x + 4y)$, if $x^2 + y^2 = 25$ is given by

23. The number of points at which the function $f(x) = (x - |x|)^2 (1 - x + |x|)^2$ is not differentiable in the interval $(-3, 4)$ is

24. If $f(x)=\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$; where ω is a complex cube root of unity. Then find the value of $\int_{-\pi/2}^{\pi/2} f(x)dx$ ________.

25. If $f(x)=\sqrt[3]{\frac{9}{\log_2(3-2x)}-1}$, then the value of a which satisfies $f^{-1}(2a - 4) = 1/2$, is given by

26. If the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$. Then the value of $\alpha + \beta$ is

27. Let f and g are the two functions symmetric about the line $x = a$ and $h(x)$ be a function such that $3h(x) - 4h(2a - x) = 5$. Then the value of $\frac{-\int_0^{2a} f(x)\cdot g(x)\cdot h(x)\,dx}{\int_0^{2a} f(x)\cdot g(x)\,dx}$ is

28. Number of solutions of the equation $[\cos x] = \{\sin\{x\}\}$ in $[0, 2\pi]$ are

29. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ and $f(0) = 1$ and g be a function satisfying $f(x) + g(x) = x^2$. If the value of $\int_0^1 f(x)g(x)\,dx - e + \frac{e^2}{2}$ is k, then the value of $|2k|$ is

30. The absolute value of intercept of the common tangent to the curves $y^2 = 8x$ and $xy + 1 = 0$ on the axis of x is

SOLUTIONS

1. (c) : Let α, β be the roots of $x^2 - bx + c = 0$, then $\alpha + \beta = b$

$\Rightarrow$ One root is 2, as roots are prime and b is odd integer

[$\because$ α and β both simultaneously can't be odd, so one root must be odd and other even]

$\therefore$ $f(2) = 0 \Rightarrow 2b - c = 4$ and $b + c = 35$ (given)

So, $b = 13$, $c = 22$

$\therefore$ Minimum value $= f\left(\frac{13}{2}\right) = -\frac{81}{4}$

2. (c) : Required coefficient of x^6 is given by

$$2\left\{\frac{1}{1!\times 5!} + \frac{1}{2!\times 4!}\right\} + \frac{1}{(3!)^2}$$

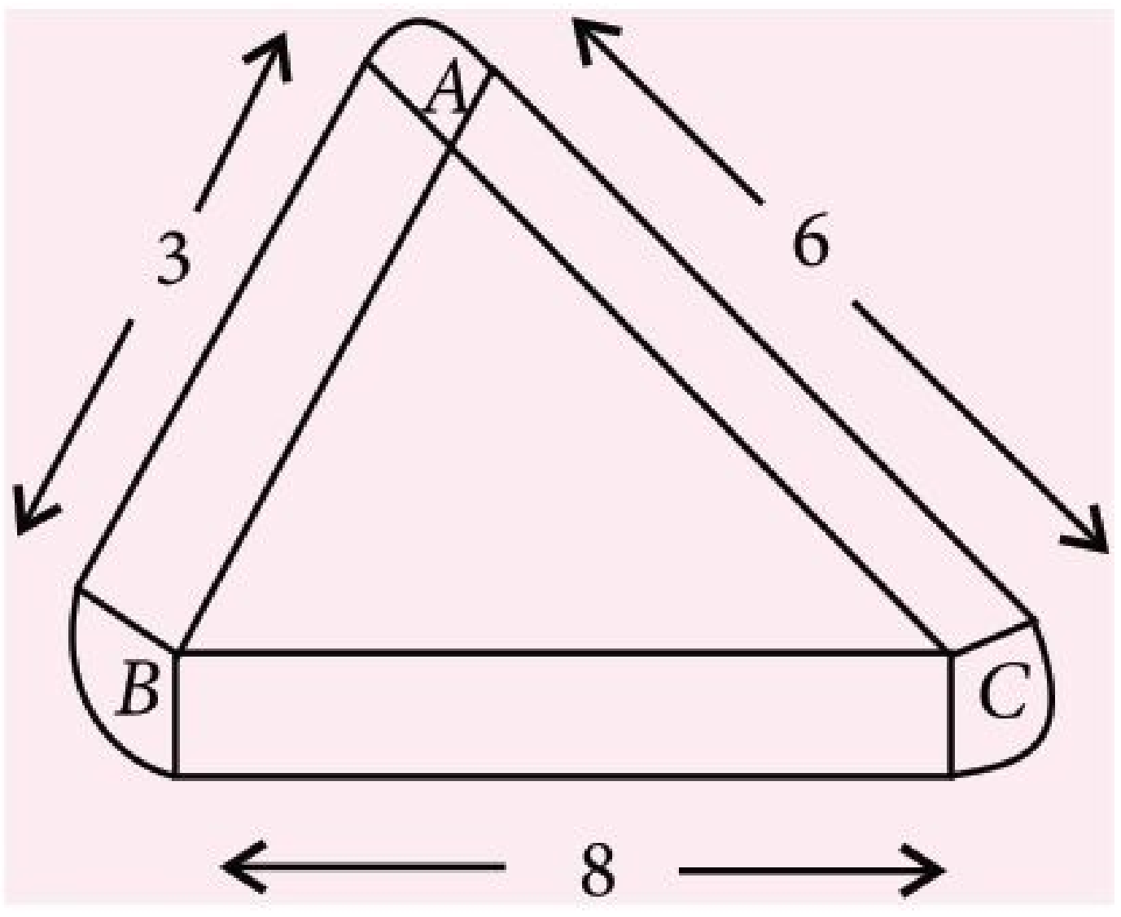
$$= \frac{1}{6!}\{{}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5\} = \frac{2^6 - 2}{6!} = \frac{31}{360}$$

3. (c) : As $f(x) = 1/2$

$\Rightarrow$ $x - [x] = 1$, which is not true

as $x - [x] \in [0, 1)$ *i.e.*, given set is the empty set.

4. (c) :



Distance travelled by man is

$$3 + 6 + 8 + (\pi - A) \times 1 + (\pi - B) \times 1 + (\pi - C) \times 1$$
$$= 17 + 3\pi - (A + B + C) = 17 + 2\pi$$

5. (b): The equation of pair of tangents from (x_1, y_1) on curve $\equiv SS_1 = T^2$

$$\therefore \left(\frac{x^2}{12} - \frac{y^2}{7} - 1\right)\left(\frac{1^2}{12} - \frac{2^2}{7} - 1\right) = \left(\frac{1\cdot x}{12} - \frac{y(2)}{7} - 1\right)^2$$

$$\Rightarrow \left(\frac{x^2}{12} - \frac{y^2}{7} - 1\right)\left(\frac{7 - 48 - 84}{84}\right) = \left(\frac{7x - 24y - 84}{84}\right)^2$$

On solving, we have

$$(7x^2 - 12y^2 - 84)(7 - 48 - 84) = (7x - 24y - 84)^2$$

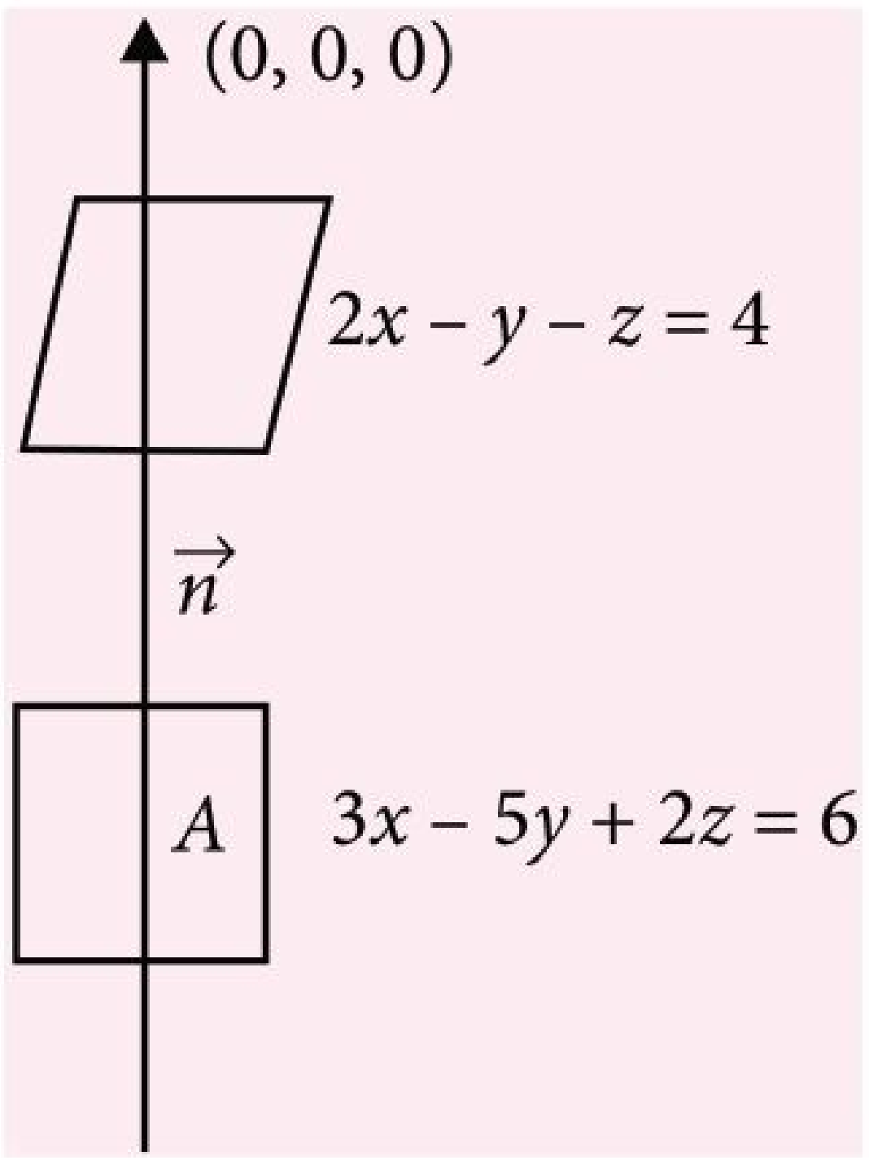
$\Rightarrow$ $924x^2 - 924y^2 - 336xy + \alpha x + \beta y + c = 0$

$\because$ Coefficient of x^2 + coefficient of $y^2 = 0$

$\therefore$ Angle subtended is $\pi/2$

6. (d) : Vector perpendicular to $2x - y - z = 4$ is given by

$\vec{n} = 2\hat{i} - \hat{j} - \hat{k}$



Also, line is collinear with $\vec{n}$.

So, equation of line is

$$\vec{r} = \lambda(2\hat{i} - \hat{j} - \hat{k}) \quad \ldots\text{(i)}$$

Now, line (i) meets the plane

$$\vec{r}\cdot(3\hat{i} - 5\hat{j} + 2\hat{k}) = 6$$

So, $\lambda(6 + 5 - 2) = 6 \Rightarrow \lambda = 2/3$

Hence, P.V. of the point A is $\frac{2}{3}(2\hat{i} - \hat{j} - \hat{k})$

i.e. $\left(\frac{4}{3}, \frac{-2}{3}, \frac{-2}{3}\right)$

7. (d): $\because$ $f'(x) = \frac{1}{1 + x^3}$

Also, $f(g(x)) = \int_0^{g(x)} \frac{dt}{1 + t^3}$ $\quad (\because g = f^{-1})$

$\Rightarrow$ $x = \int_0^{g(x)} \frac{dt}{1 + t^3}$

Now, $1 = \frac{1}{1 + (g(x))^3} g'(x) \Rightarrow g'(x) = 1 + (g(x))^3$

$$= \frac{1}{f'(g(x))}$$

8. (a)

9. (d): $\int_0^{n\pi + t} |\sin x|\,dx = \int_0^{n\pi} |\sin x|\,dx + \int_{n\pi}^{n\pi + t} |\sin x|\,dx$

$$= 2n + (-\cos x)_{n\pi}^{n\pi + t} = 2n + \cos(n\pi) - \cos(n\pi + t)$$

$$= \begin{cases} 2n + 1 - \cos t; & n = \text{even} \\ 2n - 1 + \cos t; & n = \text{odd} \end{cases}$$

10. (b): $\because$ $B = A + A^2 + A^3 + A^4 = \frac{A(I - A^4)}{I - A} = \frac{A - A^5}{I - A}$

Now, $A^5 = \begin{bmatrix} \cos 5\alpha & \sin 5\alpha \\ -\sin 5\alpha & \cos 5\alpha \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$

11. (c) : $P(E) = P(A)P(B') + P(A')P(B) + P(A)\cdot P(B)$

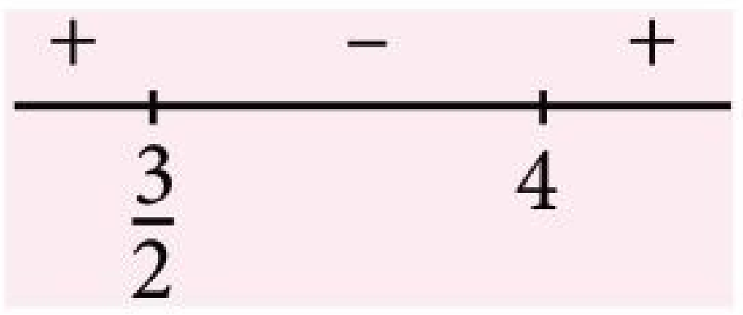
$$= \left(\frac{1}{3}\times\frac{4}{5}\right) + \left(\frac{2}{3}\times\frac{1}{5}\right) + \left(\frac{1}{3}\times\frac{1}{5}\right) = \frac{4}{15} + \frac{2}{15} + \frac{1}{15} = \frac{7}{15}$$

12. (d): Using LMVT,

$$f'(c) = \frac{f(b) - f(a)}{b - a} \Rightarrow \frac{1}{c} = \frac{1 - 0}{e - 1}$$

$\therefore$ The point on the curve is $(e - 1, \ln(e - 1))$

13. (b): $f(x) = 4x^3 - 33x^2 + 72x + 6$



$\Rightarrow\ f'(x) = 12x^2 - 66x + 72$
$= 6(2x^2 - 11x + 12)$
$= 6(2x - 3)(x - 4)$

$\Rightarrow$ $f(x)$ is increasing in $(-\infty, 3/2) \cup (4, \infty)$ and decreasing in $(3/2, 4)$

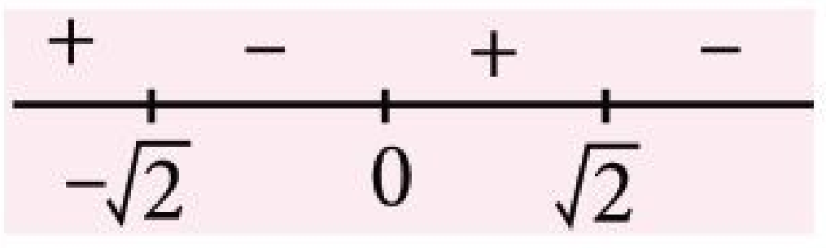
$\Rightarrow\ f(4) = 256 - 528 + 288 + 6 > 0$

$$\Rightarrow\ f\left(\frac{3}{2}\right)=\frac{27}{2}-\frac{297}{4}+108+6>0$$

Also, $f(0) = 6 > 0$

$\Rightarrow$ $f(x)$ has one negative real root and other two roots are complex.

14. (d): $f(x) = x^4 e^{-x^2}$



$\Rightarrow\ f'(x) = 4x^3e^{-x^2} - 2x^5e^{-x^2}$

$\Rightarrow\ 2x^3 e^{-x^2}(2 - x^2) = 0$

$\Rightarrow\ x = 0, \pm\sqrt{2}$

$\therefore$ $f(x)$ has maxima at $x = \pm\sqrt{2}$ and minima at $x = 0$

15. (c) : Let $\frac{1}{2r+1}=\lambda$ and $\frac{1}{2r+5}=\mu$

$$\Rightarrow\ \sum_{r=1}^{n}(3x-\lambda)(2x-\mu)=0$$

$$\Rightarrow\ \sum_{r=1}^{n}6x^2-x(3\mu+2\lambda)+\lambda\mu=0$$

Since $\sum_{r=1}^{n}\lambda\mu=\sum_{r=1}^{n}\frac{1}{(2r+1)(2r+5)}$

$$\Rightarrow\ \frac{1}{4}\sum_{r=1}^{n}\left(\frac{1}{2r+1}-\frac{1}{2r+5}\right)=\frac{1}{4}\left[\frac{1}{3}-\frac{1}{2n+5}\right]=\frac{n+1}{6(2n+5)}$$

$\therefore$ The product of roots of the equation $=\frac{n+1}{36n(2n+5)}$

16. (a) : $f(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}$

$$f'(x)=\frac{(e^x+e^{-x})(e^x+e^{-x})-(e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$$

$$=\frac{(e^x+e^{-x})^2-(e^x-e^{-x})^2}{(e^x+e^{-x})^2}$$

$$=\frac{2e^x\cdot 2e^{-x}}{(e^x+e^{-x})^2}=\frac{4}{(e^x+e^{-x})^2}>0,\ \forall\ x\in R$$

$\Rightarrow$ f is increasing function for all x in R

$\Rightarrow$ f is one-one.

As $x\to\infty$, $f(x)=\frac{1-e^{-2x}}{1+e^{-2x}}\to 1$

and $x\to-\infty$, $f(x)=\frac{e^{2x}-1}{e^{2x}+1}\to -1$

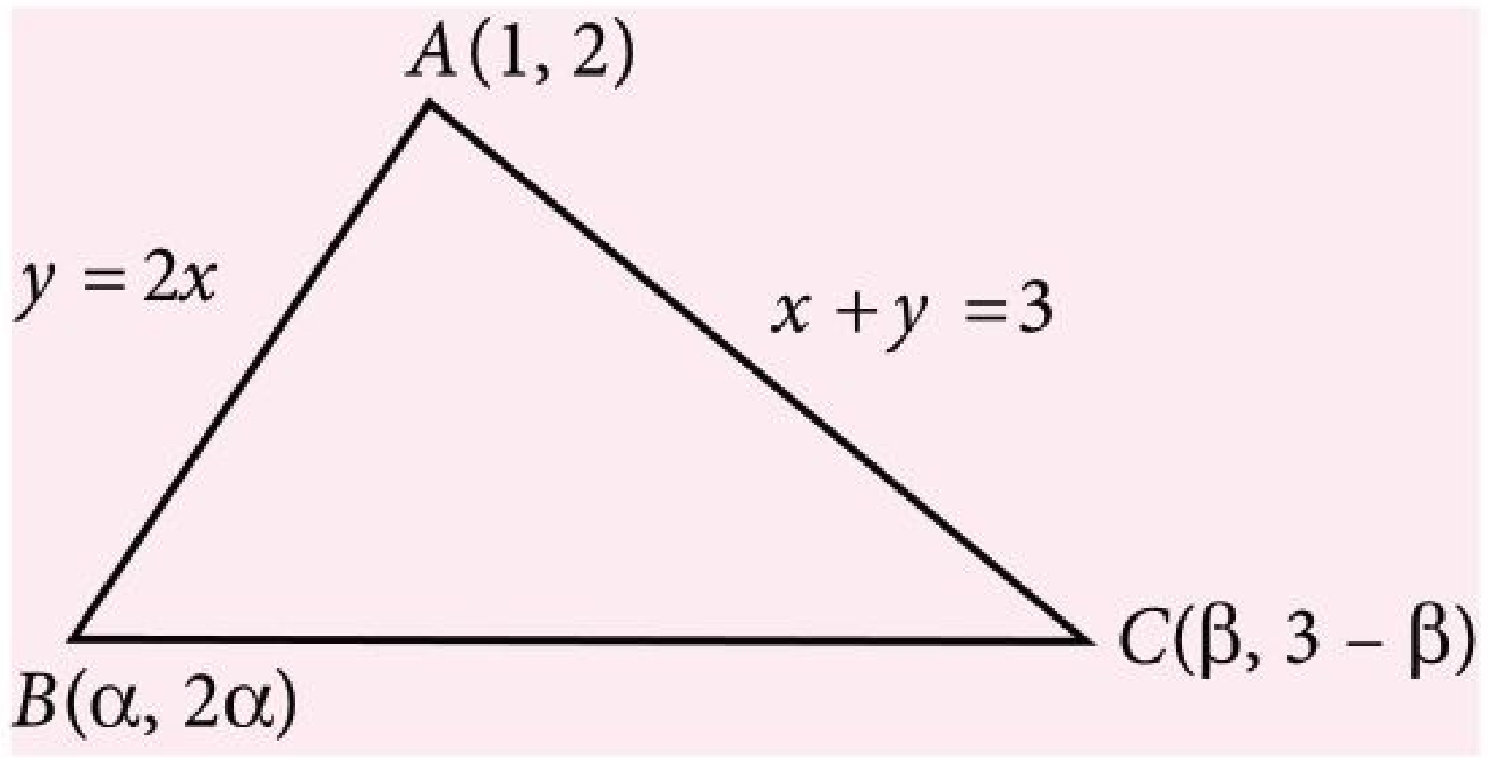
$\therefore$ Range is $(-1, 1)$ $\Rightarrow$ f is onto.

17. (b): The lines $2x - y = 0$, $x + y = 3$ meet at $A(1, 2)$.

Any point on $2x - y = 0$ is $B(\alpha, 2\alpha)$

Any point on $x + y = 3$ is $C(\beta, 3 - \beta)$

Centroid is $(2, 3)$



$$\therefore\ 2=\frac{1+\alpha+\beta}{3},\ 3=\frac{2+2\alpha+3-\beta}{3}$$

or $\alpha + \beta = 5,\ 2\alpha - \beta = 4 \Rightarrow \alpha = 3,\ \beta = 2$

$\therefore$ $B = (3, 6)$ and $C = (2, 1)$

Equation of BC is $\frac{y-1}{x-2}=\frac{6-1}{3-2}=5$ or $5x - y - 9 = 0$.

18. (a) : $A^TA = I \Rightarrow \det(A^TA) = \det I$

$\Rightarrow\ \det A^T \cdot \det A = 1 \Rightarrow (\det A)^2 = 1 \Rightarrow \det A = \pm 1$

$$\det A=\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}=3abc-(a^3+b^3+c^3)$$

$$=-\frac{(a+b+c)}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]<0$$

($\because$ a, b, c are positive)

$\Rightarrow\ \det A = -1 \Rightarrow 3abc - (a^3 + b^3 + c^3) = -1$

$\Rightarrow\ a^3 + b^3 + c^3 = 3abc + 1 = 3 \times 1 + 1 = 4.$

19. (c) : $f'(x)=\frac{\sin x-x\cos x}{\sin^2 x}=\frac{\cos x}{\sin^2 x}(\tan x-x)>0$

since $\tan x > x$.

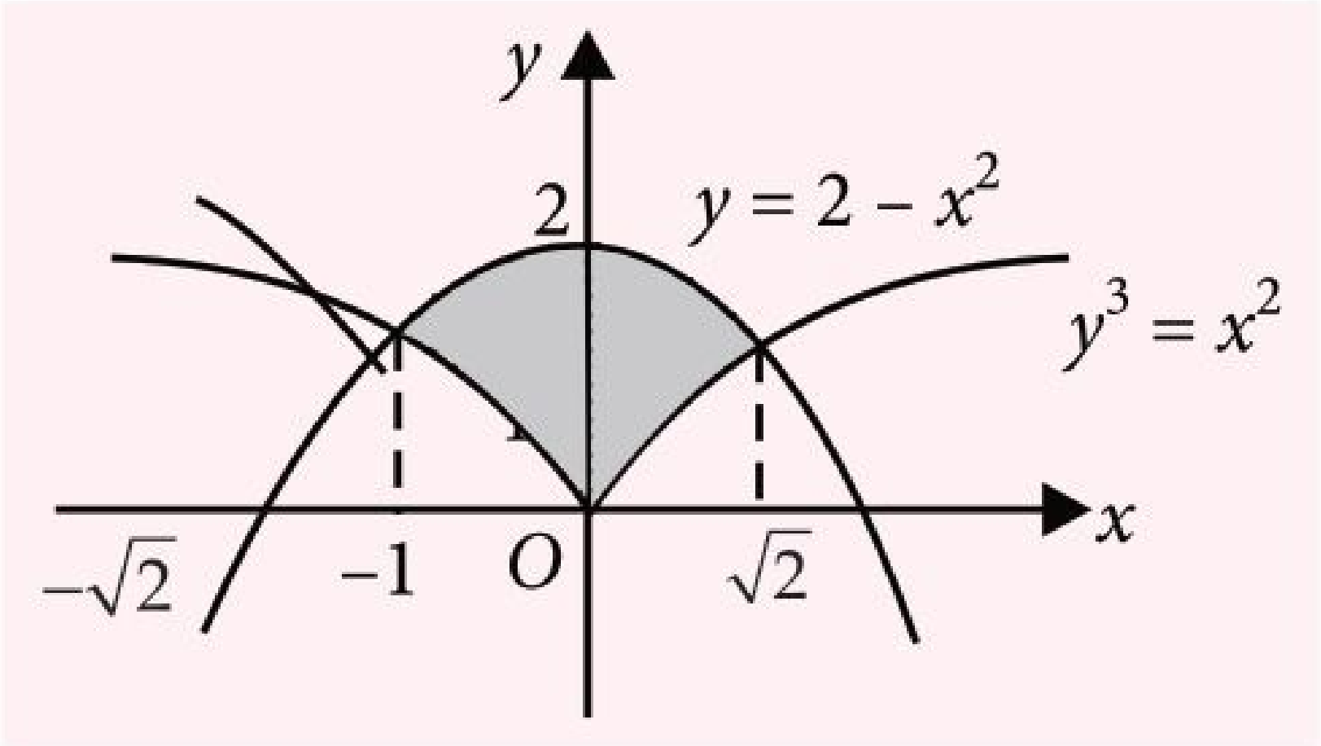
$$g'(x)=\frac{\tan x-x\sec^2 x}{\tan^2 x}=\frac{\sin 2x-2x}{2\sin^2 x}<0$$

since $\sin x < x$.

$\therefore$ f increases and g decreases.

20. (c) : Eliminating y, $x^2 = 1 \Rightarrow x = \pm 1$

$\therefore$ Required area



$$=2\int_0^1\left(2-x^2-x^{\frac{2}{3}}\right)dx$$

$$=2\left[2-\frac{1}{3}-\frac{3}{5}\right]=\frac{32}{15}.$$

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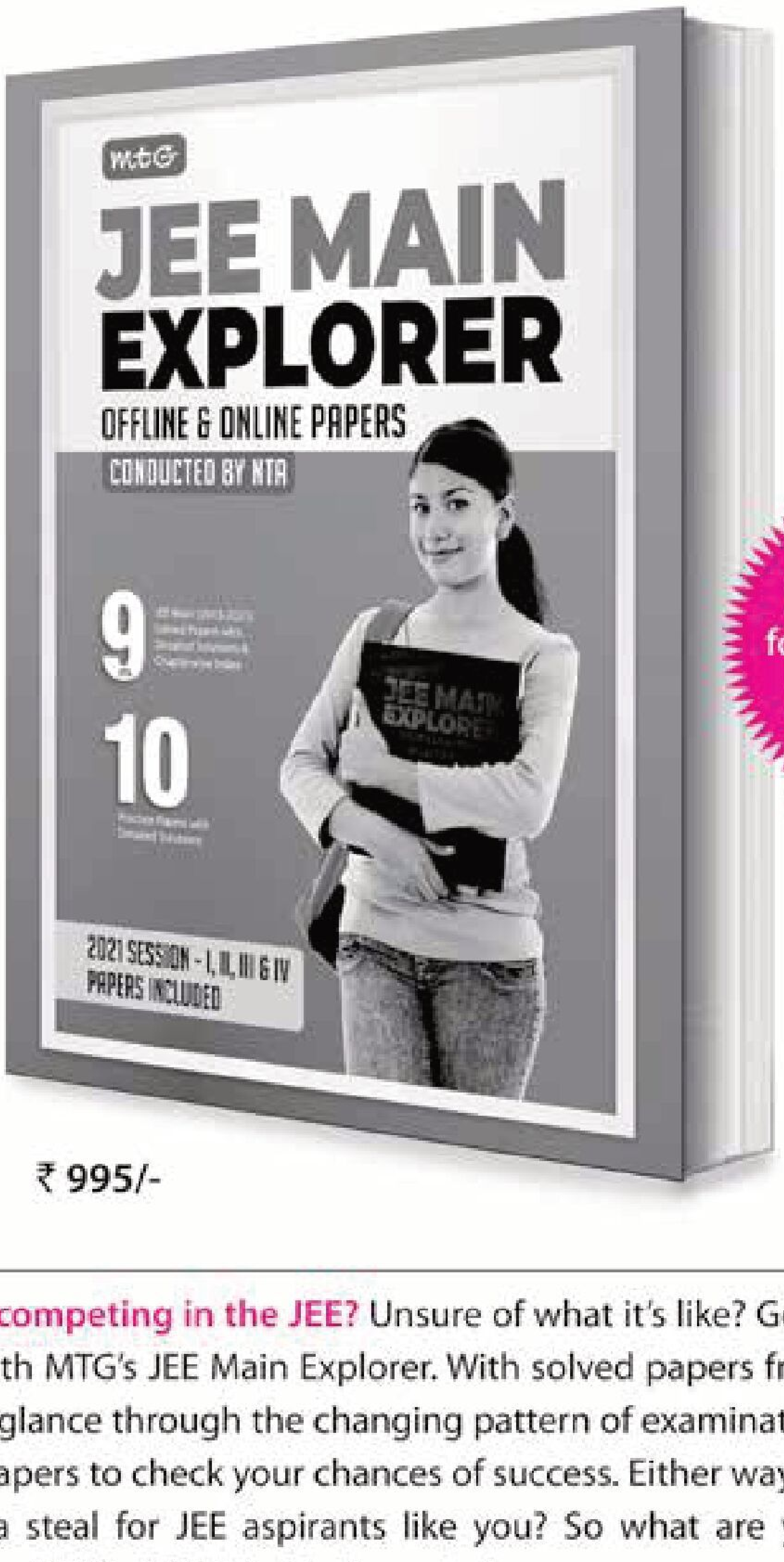
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21. (28) : As, $\alpha^6=1$

$$\Rightarrow \alpha=\left(\cos\frac{2\pi}{6}+i\sin\frac{2\pi}{6}\right)^m \text{, where } m \longrightarrow 0 \text{ to } 5$$

Taking any one of its roots, $\alpha=\cos\frac{2\pi}{6}+i\sin\frac{2\pi}{6}$

Now, $\sum\limits_{r=1}^{6} {}^6C_r \cdot \alpha^r = \alpha x$

$$\Rightarrow \alpha x=(1+\alpha)^6-1=\left\{2\cos\frac{\pi}{6}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right\}^6-1$$

$$= -27 - 1 = -28$$

$\Rightarrow$ $|\alpha x| = 28 \Rightarrow |x| = 28$

22. (2): Let $x = 5\cos\theta$, $y = 5\sin\theta$

So, given expression $= \log_5\{5(3\cos\theta + 4\sin\theta)\}$

As $3\cos\theta + 4\sin\theta \le 5$

$\Rightarrow$ Maximum value $= \log_5(25) = 2$

23. (0): Given $f(x)$ can be written as

$$f(x)=\begin{cases}16x^4-16x^3+4x^2, & x<0\\ 0, & x\ge 0\end{cases}$$

Clearly, $f(x)$ is continuous as well as derivable $\forall\, x \in R$.

24. (0): We have,

$$f(x)=\begin{vmatrix}x+1 & \omega & \omega^2\\ \omega & x+\omega^2 & 1\\ \omega^2 & 1 & x+\omega\end{vmatrix} \qquad C_1 \longrightarrow C_1+C_2+C_3$$

$$\Rightarrow f(x)=x\begin{vmatrix}1 & \omega & \omega^2\\ 1 & x+\omega^2 & 1\\ 1 & 1 & x+\omega\end{vmatrix}$$

$R_2 \longrightarrow R_2 - R_1$, $R_3 \longrightarrow R_3 - R_1$

$$f(x)=x\begin{vmatrix}1 & \omega & \omega^2\\ 0 & x+\omega^2-\omega & 1-\omega^2\\ 0 & 1-\omega & x+\omega-\omega^2\end{vmatrix}$$

$= x[(x + \omega^2 - \omega)(x + \omega - \omega^2) - (1 - \omega^2)(1 - \omega)]$

$= x[x^2 - (\omega + \omega^2 - 2) - (1 - \omega - \omega^2 + 1)] = x^3$

$\Rightarrow$ $f(x)$ is an odd function.

$$\therefore \int_{-\pi/2}^{\pi/2} f(x)dx=0$$

25. (3): Given $f^{-1}(2a-4)=\frac{1}{2}$

$$\therefore f\left(\frac{1}{2}\right)=2a-4$$

Put $x = 1/2$ in $f(x)=\sqrt[3]{\frac{9}{\log_2(3-2x)}-1}$

We have, $f\left(\frac{1}{2}\right)=\sqrt[3]{9-1}=2$

$\therefore$ $2a - 4 = 2 \Rightarrow a = 3$

26. (1): The required conditions are

$3 - 15 - 2\alpha = 0$ ... (i)

and $2 + 3 + 2\alpha + \beta = 0$ ... (ii)

$\Rightarrow$ $\alpha = -6$ and $\beta = 7$

$\therefore$ $\alpha + \beta = 1$

27. (5): $\because f(a - x) = f(a + x)$

or, $f(x + 2a) = f(-x) \Rightarrow f(x) = f(2a - x)$

Similarly, $g(2a - x) = g(x)$

Now, let $I=\int_0^{2a} f(x)\cdot g(x)\cdot h(x)dx$

$$=\int_0^{2a} f(x)\cdot g(x)\cdot h(2a-x)dx$$

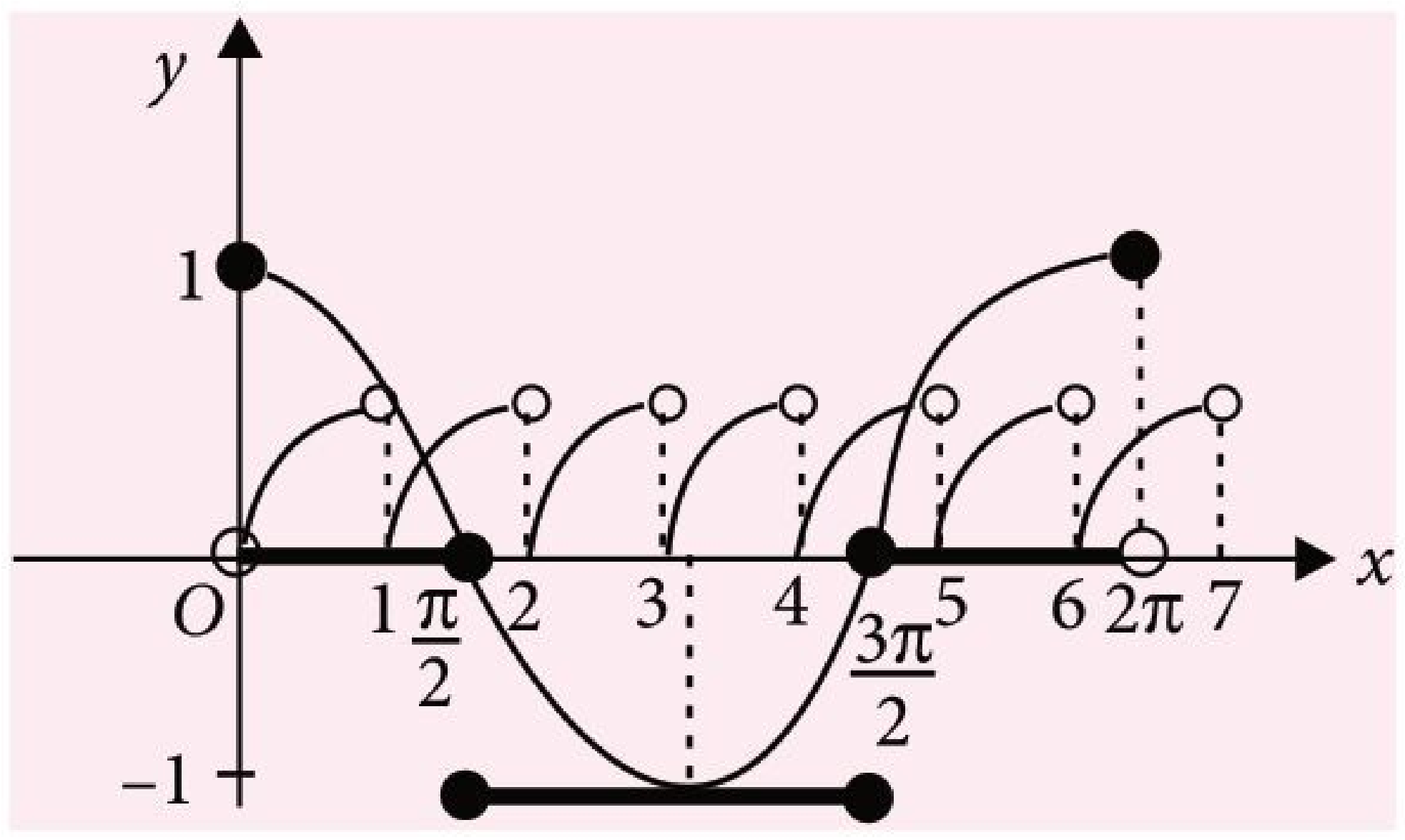
or, $4I=\int_0^{2a} f(x)\cdot g(x)\cdot(3h(x)-5)dx$

$$=3I-5\int_0^{2a} f(x)\cdot g(x)dx$$

$$\therefore I=-5\int_0^{2a} f(x)\cdot g(x)dx$$

$$\Rightarrow \frac{\int_0^{2a} f(x)\cdot g(x)\cdot h(x)dx}{\int_0^{2a} f(x)\cdot g(x)dx}=-5$$

28. (3): From graph it can be seen that number of solutions is 3.



MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(b)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(c)
6.	(b)	**7.**	(a, d)	**8.**	(a, b, c)	**9.**	(c, d)	**10.**	(a, b)
11.	(b, c)	**12.**	(c, d)	**13.**	(d)	**14.**	(c)	**5.**	(a)
16.	(a)	**17.**	(5)	**18.**	(8)	**19.**	(2)	**20.**	(1)

29. (3) : $\because f'(x) = f(x)$ and $f(0) = 1 \Rightarrow f(x) = e^x$

$\therefore \quad g(x) = x^2 - e^x$

Now, $\int_0^1 f(x)g(x)\,dx = \int_0^1 e^x(x^2 - e^x)dx = e - \frac{e^2}{2} - \frac{3}{2}$

$$\Rightarrow \int_0^1 f(x)g(x)dx - e + \frac{e^2}{2} = \frac{-3}{2} \Rightarrow k = -\frac{3}{2} \Rightarrow |2k| = 3$$

30. (2) : Equation of tangent of $y^2 = 8x$ is

$$y = mx + \frac{2}{m} \quad \text{... (i)}$$

and $xy + 1 = 0$ ... (ii)

From (i) and (ii)

$$x\left(mx + \frac{2}{m}\right) + 1 = 0 \quad \text{or} \quad mx^2 + \frac{2}{m}x + 1 = 0$$

$$\Rightarrow \quad D = 0 \Rightarrow \frac{4}{m^2} = 4m \Rightarrow m^3 = 1$$

$\therefore \quad m = 1$

$\therefore$ common tangent is $y = x + 2$

$\therefore$ x-intercept = -2

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PUBLIC NOTICE JEE MAIN 2022

The Department of Higher Education, Ministry of Education, Government of India has entrusted the responsibility of conducting the Joint Entrance Examination (JEE Main) to the NTA from 2019 onwards.

The Joint Entrance Examination **(JEE (Main)** comprises **two papers. Paper 1** is conducted for admission to Undergraduate Engineering Programs **(B.E./B. Tech)** at NIT's, IITs, other Centrally Funded Technical Institutions (CFTIs), Institutions/Universities funded/recognized by participating State Governments. It is also an eligibility test for JEE (Advanced), which is conducted for admission to IITs. **Paper 2** is conducted for admission to **B. Arch and B. Planning** courses in the Country.

The JEE (Main) - 2022 will be conducted in **02 (two) sessions** for admissions in the next academic session (2022-23). The details are given below :

Paper	Subjects	Section A	Section B	Mode of Examination	Timing of The Examination (IST)	
					First Shift	Second Shift
Paper 1 **(B.E. / B. Tech.)**	Mathematics	20*	10*	**Computer Based Test (CBT) mode**	**9:00 a.m. to 12:00 p.m.**	**3:00 p.m. to 6:00 p.m.**
	Physics	20*	10*			
	Chemistry	20*	10*			
	Total	**90**				
Paper 2A **(B. Arch.)**	Mathematics - Part I	20*	10*	**Computer Based Test (CBT) mode except** Drawing Test (Part-III) in pen and paper (offline) mode, to be attempted on drawing sheet of A4 size	**9:00 a.m. to 12:00 p.m.**	**3:00 p.m. to 6:00 p.m.**
	Aptitude Test - Part II	50				
	Drawing Test - Part III	02				
	Total	**82**				
Paper 2B **(B. Planning)**	Mathematics - Part I	20*	10*	**Computer Based Test (CBT) mode**	**3:00 p.m. to 6:00 p.m.**	
	Aptitude Test - Part II	50				
	Planning - Part III	25				
	Total	**105**				

*For Paper 1 and Part-I of Paper 2, each Subject will have **two sections.** Section A will be of Multiple-Choice Questions (MCQs) and Section B will contain Questions whose answers are to be filled in as a numerical value. In Section B, candidates have to attempt any **05 (five) questions out of 10. There will be negative marking for both Section A and Section B.** For each question in Section B, **enter the correct integer value** of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. **For Section B, the answer should be rounded off to the nearest Integer.**

Medium of the Question Papers: Drawing from the National Education Policy (NEP), the JEE (Main) 2022 will be conducted in *Assamese, Bengali, Kannada, Malayalam, Marathi, Odia, Punjabi, Tamil, Telugu, Urdu in addition to Hindi, English and Gujarati.*

Dates of Examinations :

Session-1	Session-2
21, 24, 25, 29 April and 1, 4 May	24, 25, 26, 27, 28 and 29 May 2022

Exam Dates
Session-1
21, 24, 25, 29 April and 1, 4 May
Session-2
24, 25, 26, 27, 28 and 29 May 2022

JEE Main 2022

MOCK TEST PAPER

SECTION - A

MULTIPLE CHOICE QUESTIONS

1. In a triangle, sides $a, b, c \in$ A.P., then
(a) $\angle B \le 60°$
(b) triangle is right angle triangle
(c) triangle is obtuse angle triangle
(d) None of these

2. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, then $(\vec{a}+\vec{b}+\vec{c})\cdot[(\vec{a}+\vec{b})\times(\vec{a}+\vec{c})]$ is
(a) 0 (b) $2[\vec{a}\ \vec{b}\ \vec{c}]$
(c) $-[\vec{a}\ \vec{b}\ \vec{c}]$ (d) $[\vec{a}\ \vec{b}\ \vec{c}]$

3. The denominator of a fraction number is greater than 16 of the square of numerator, then least value of the number is
(a) $-1/4$ (b) $-1/8$ (c) $1/16$ (d) $-1/16$

4. Let $f(x) = a + bx + cx^2$, $c > 0$, $D < 0$ then area bounded by coordinates axes, line $x = 2$ & $y = f(x)$ is equal to
(a) $\frac{1}{2}[f(0) + 4f(1)]$
(b) $\frac{1}{3}[4f(1) + f(2)]$
(c) $\frac{1}{2}[f(0) + 4f(1) + f(2)]$
(d) $\frac{1}{3}[f(0) + 4f(1) + f(2)]$

5. The equation of one of the lines parallel to $4x - 3y = 5$ and at a unit distance from the point $(-1, -4)$ is
(a) $3x + 4y - 3 = 0$ (b) $3x + 4y + 3 = 0$
(c) $4x - 3y + 3 = 0$ (d) $4x - 3y - 3 = 0$

6. The radius of a circle passing through the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and centred at (0, 4) is
(a) 5 (b) 4
(c) 3 (d) None of these

7. If $x \ln x \frac{dy}{dx} + y = 2\ln x$, $y(e) = 2$, then $y(e^2) =$
(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{5}{2}$

8. If $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ satisfies $A^4 = kA^T$, then find the value of k.
(a) 1 (b) 4 (c) 8 (d) 10

9. The range of the function $f(x) = 9^x - 3^x + 1$ is
(a) $(-\infty, \infty)$ (b) $(-\infty, 0)$
(c) $(0, \infty)$ (d) $\left[\frac{3}{4},\infty\right)$

10. Let $S = \frac{4}{19}+\frac{44}{19^2}+\frac{444}{19^3}+.....\infty$. Then S is equal to
(a) $\frac{38}{81}$ (b) $\frac{4}{19}$ (c) $\frac{36}{171}$ (d) $\frac{1}{19}$

11. Let $f(x) = \min\{\sin x, \cos x\}$ then find the area bounded by $f(x)$, x-axis and $x = 0$, $x=\frac{\pi}{2}$.
(a) $2(\sqrt{3}-1)$ sq. units (b) $\sqrt{2}(\sqrt{2}-1)$ sq. units
(c) $(\sqrt{2}-1)$ sq. units (d) $\sqrt{2}(\sqrt{2}+1)$ sq. units

12. If the vectors $\hat{i}-2\hat{j}+3\hat{k}$, $-2\hat{i}+3\hat{j}-4\hat{k}$, $\lambda\hat{i}-\hat{j}+2\hat{k}$ are coplanar, then the value of λ is equal to
(a) 0 (b) 1 (c) 2 (d) 3

13. If $iz^3 + z^2 - z + i = 0$, then $|z|$ equals
(a) 2 (b) 1
(c) 0 (d) None of these

14. $\int \frac{\log(x/e)}{(\log x)^2}dx$ is equal to

(a) $\frac{\log x}{x}+c$ (b) $\frac{x}{\log x}+c$

(c) $\frac{x}{(\log x)^2}+c$ (d) None of these

15. The domain of $\sin^{-1}[x]$ is given by

(a) $[-1, 1]$ (b) $[-1, 2)$

(c) $\{-1, 0, 1\}$ (d) None of these

16. The value of $|z|^2 + |z-3|^2 + |z-i|^2$ is minimum then z equals

(a) $1-\frac{i}{3}$ (b) $2-\frac{2}{3}i$ (c) $4+\frac{1}{3}i$ (d) $1+\frac{i}{3}$

17. $\lim\limits_{x \to 1} \frac{(2x-3)\left(\sqrt{x}-1\right)}{2x^2+x-3} =$

(a) $\frac{1}{10}$ (b) $\frac{2}{15}$ (c) $-\frac{1}{10}$ (d) $-\frac{2}{15}$

18. The area of the triangle formed by the coordinate axes and a tangent to the curve $xy = a^2$ at the point (x_1, y_1) on it is

(a) $\frac{a^2x_1}{y_1}$ (b) $\frac{a^2y_1}{x_1}$ (c) $2a^2$ (d) $4a^2$

19. Consider the system of equations $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$, $a_3x + b_3y + c_3z = 0$,

If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, then the system has

(a) more than two solutions

(b) one trivial and one non-trivial solution

(c) no solution

(d) only trivial solution (0, 0, 0)

20. If the binomial expansion of $(a+bx)^{-2}$ is $\frac{1}{4}-3x+...$, then the value of a and b are

(a) 2, 12 (b) 2, 10 (c) 1, 12 (d) 1, 15

SECTION - B

NUMERICAL TYPE

Attempt any 5 questions out of 10.

21. The number of integers greater than 6000 that can be formed by using the digits 3, 5, 6, 7 and 8 if digits are not repeated is ______ .

22. $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx = Ax\cos^{-1}x + B\sqrt{1-x^2}+c$, then $A + B =$ _______.

23. The equation of a curve whose slope at any point is thrice its abscissa and which passes through $(-1, -3)$ is $2y = \lambda(x^2 - 3)$, then the value of λ must be ______.

24. Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Then the number of bijective functions $f: A \to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to _______.

25. In a triangle ABC, $a = 9$, $b : c = 40 : 41$. If its largest area is N, then the last digit of N is ______ .

26. If the distance of the point $B(\hat{i}+2\hat{j}+3\hat{k})$ from the line which is passing through $A(4\hat{i}+2\hat{j}+2\hat{k})$ and which is parallel to the vector $\vec{c}=2\hat{i}+3\hat{j}+6\hat{k}$ is λ, then the value of $\lambda^6 + \lambda^4 + \lambda^2 + 1$ must be ______.

27. Water is dropped at the rate of 2 m^3/s into a cone of semi vertical angle 45°. If the rate at which periphery of water surface changes when the height of the water in the cone is 2 m is d m/s, then the value of $5d$ is ________.

28. The number of integral values of α for which the point $(\alpha - 1, \alpha + 1)$ lies in the larger segment of the circle $x^2 + y^2 - x - y - 6 = 0$ made by the chord whose equation is $x + y - 2 = 0$ is ______.

29. There are two possible values of A(say A_1 and A_2) in the solution of matrix equation $\begin{bmatrix} 2A+1 & -5 \\ -4 & A \end{bmatrix}^{-1}$ $\begin{bmatrix} A-5 & B \\ 2A-2 & C \end{bmatrix} = \begin{bmatrix} 14 & S \\ E & F \end{bmatrix}$, then find $-27(A_1 + A_2)$.

30. An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white, it is not replaced into urn, otherwise it is replaced along with another ball of the same colour. The process is repeated. If the probability that the third ball drawn is black is $\frac{23}{\lambda}$, then the value of λ must be _________.

SOLUTIONS

1. (a) : Given $a, b, c \in$ A.P.

$\therefore \quad 2b = a + c$

Using sine formula, we get

$$2\sin B = \sin A + \sin C = 2\sin\frac{A+C}{2}\cdot\cos\frac{A-C}{2}$$

$$2\sin\frac{B}{2} = \cos\frac{A-C}{2} \Rightarrow \sin\frac{B}{2} = \frac{1}{2}\cos\frac{A-C}{2}$$

$$\Rightarrow \quad \sin\frac{B}{2} \le \frac{1}{2} \Rightarrow \frac{B}{2} \le 30° \Rightarrow B \le 60°$$

2. **(c) :** Given, $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors.

Consider $(\vec{a}+\vec{b}+\vec{c})\cdot[(\vec{a}+\vec{b})\times(\vec{a}+\vec{c})]$

$= (\vec{a}+\vec{b}+\vec{c})\cdot[\vec{a}\times\vec{a}+\vec{a}\times\vec{c}+\vec{b}\times\vec{a}+\vec{b}\times\vec{c}]$

$=(\vec{a}+\vec{b}+\vec{c})\cdot[\vec{a}\times\vec{c}+\vec{b}\times\vec{a}+\vec{b}\times\vec{c}]$

$=[\vec{a}\,\vec{a}\,\vec{c}]+[\vec{a}\,\vec{b}\,\vec{a}]+[\vec{a}\,\vec{b}\,\vec{c}]+[\vec{b}\,\vec{a}\,\vec{c}]+[\vec{b}\,\vec{b}\,\vec{a}]+[\vec{b}\,\vec{b}\,\vec{c}]+[\vec{c}\,\vec{a}\,\vec{c}]$
$+[\vec{c}\,\vec{b}\,\vec{a}]+[\vec{c}\,\vec{b}\,\vec{c}]$

$=[\vec{a}\,\vec{b}\,\vec{c}]+[\vec{b}\,\vec{a}\,\vec{c}]+[\vec{c}\,\vec{b}\,\vec{a}]=-[\vec{a}\,\vec{b}\,\vec{c}]$

3. **(b):** The function $f(x) = \dfrac{x}{x^2+16}$

$$\Rightarrow \quad f'(x) = \frac{(x^2+16)\cdot 1 - x\cdot(2x)}{(x^2+16)^2}$$

$$= \frac{x^2+16-2x^2}{(x^2+16)^2} = \frac{16-x^2}{(x^2+16)^2} \qquad \text{...(i)}$$

Put $f'(x) = 0 \Rightarrow 16 - x^2 = 0 \Rightarrow x = 4, -4$

Again, $f''(x) = \dfrac{(x^2+16)^2(-2x)-(16-x^2)\,2(x^2+16)2x}{(x^2+16)^4}$

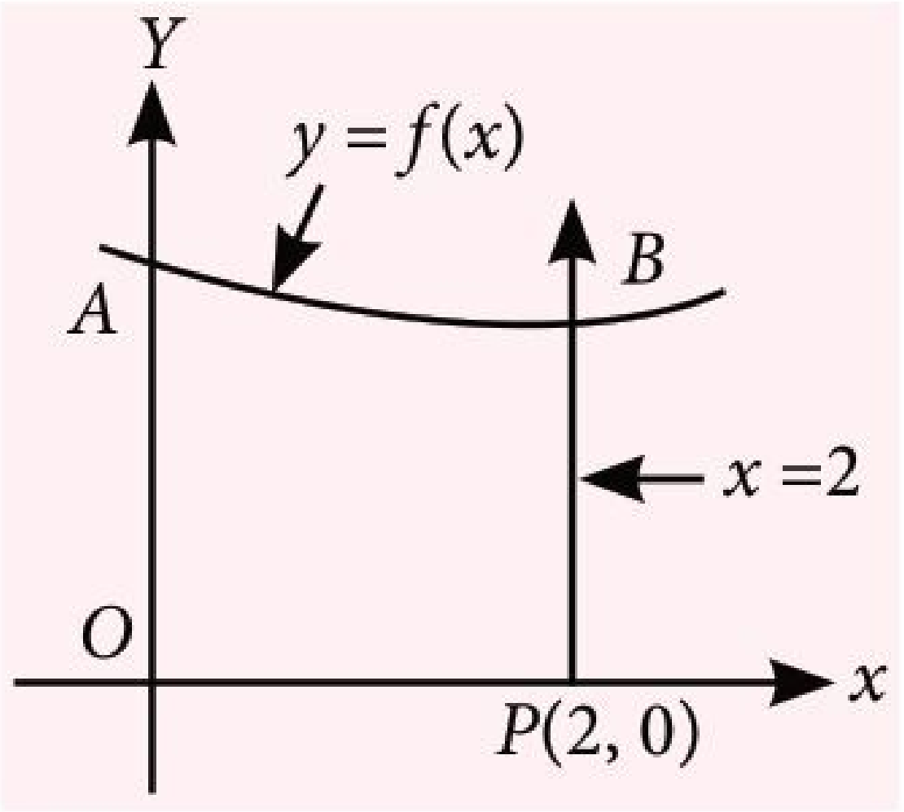
At $x = 4, f''(x) < 0$ and at $x = -4, f''(x) > 0$

$\therefore$ At $x = -4$, $f(x)$ has least value

and least value of $f(x) = \dfrac{-4}{16+16} = -\dfrac{1}{8}$.

4. **(d):** Area of $OABP = \int_0^2 y\,dx = \int_0^2 (a+bx+cx^2)dx$

$= 2a + 2b + \dfrac{8}{3}c = \dfrac{1}{3}\,[6a + 6b + 8c]$



Again, $f(x) = a + bx + cx^2$

$\Rightarrow f(0) = a, f(1) = a + b + c,$

$f(2) = a + 2b + 4c$

Now, $\dfrac{1}{3}\,[f(0) + 4f(1) + f(2)]$

$= \dfrac{1}{3}\,[a + 4(a + b + c) + (a + 2b + 4c)]$

$= \dfrac{1}{3}\,[6a + 6b + 8c] = \int_0^2 y\,dx$

5. **(d):** Equation of any line parallel to $4x - 3y = 5$ is $4x - 3y + \lambda = 0$

Let length of the perpendicular from $(-1, -4)$ to $4x - 3y + \lambda = 0$ is 1. Then

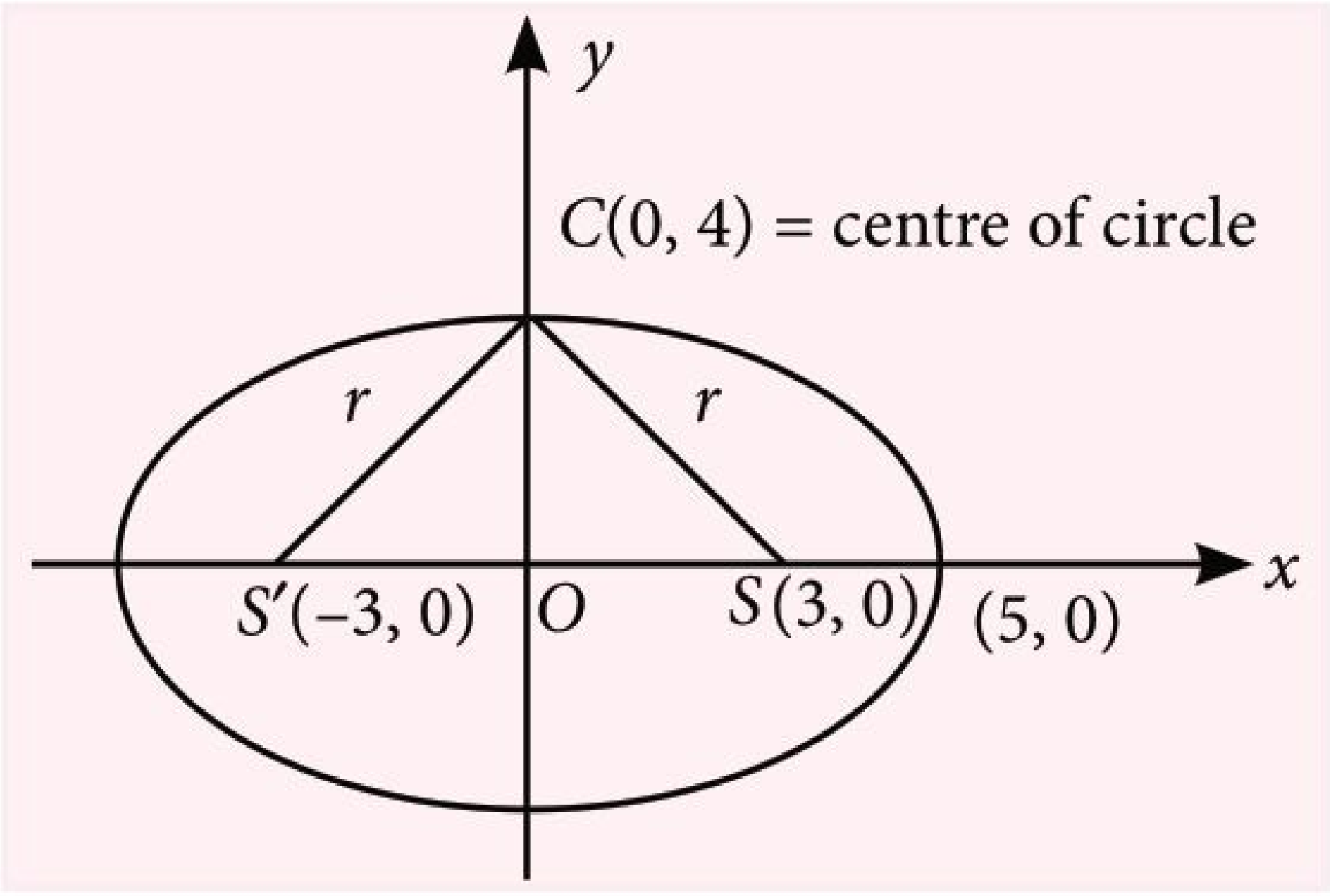
$$\left|\frac{4(-1)-3(-4)+\lambda}{\sqrt{4^2+3^2}}\right| = 1 \Rightarrow |8+\lambda| = 5$$

$\Rightarrow 8 + \lambda = 5$ or $8 + \lambda = -5 \Rightarrow \lambda = -3$, or $\lambda = -13$

$\therefore$ Required equation is $4x - 3y - 3 = 0$

or $4x - 3y - 13 = 0$

6. **(a) :** Given, $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$, comparing it with equation $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, we have $a > b$ *i.e.*, x-axis is major axis of the ellipse.



$\therefore$ Foci $= (\pm\, ae, 0)$

Now $b^2 = a^2(1 - e^2)$, where

$a^2 = 25, b^2 = 16$

$\Rightarrow 16 = 25(1 - e^2) \Rightarrow e = \dfrac{3}{5}$

$\therefore (\pm ae, 0) = (\pm 3, 0)$

$\therefore r$ = radius of circle = Distance from centre of circle to one of the foci $= CS = CS' = \sqrt{(0-3)^2+(4-0)^2} = 5$ units

$\therefore r = 5$ units

7. **(d):** $\dfrac{dy}{dx}+\dfrac{y}{x\ln x} = \dfrac{2}{x}$

It is a linear differential equation with

I.F. $= e^{\int \frac{dx}{x\ln x}} = \ln x$

$\therefore$ Solution is $y\cdot\ln x = 2\int\dfrac{\ln x}{x}dx = (\ln x)^2 + c$

Now, $x = e, y = 2 \Rightarrow 2 = 1 + c \Rightarrow c = 1$

$\Rightarrow y = \ln x + \dfrac{1}{\ln x} \;\therefore\; y(e^2) = 2+\dfrac{1}{2} = \dfrac{5}{2}$.

8. **(c) :** We have, $A = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} \Rightarrow A^T = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

$\therefore A = A^T$

Now, $A^2 = A\cdot A = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} = \begin{bmatrix}1+1 & 1+1\\1+1 & 1+1\end{bmatrix}$

$= \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix} = 2\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} = 2A$

$\therefore A^3 = 2A\cdot A = 2A^2 = 2\cdot 2A = 4A$

So, $A^4 = 4A\cdot A = 4A^2 = 4\cdot 2A = 8A = 8A^T \quad [\because A = A^T]$

So, $k = 8$

9. **(d):** Let $y = f(x) = 3^{2x} - 3^x + 1 = (3^x)^2 - 3^x + 1$

$= \left(3^x-\dfrac{1}{2}\right)^2 + 1 - \dfrac{1}{4} = \left(3^x-\dfrac{1}{2}\right)^2 + \dfrac{3}{4}$

Since, $\left(3^x-\dfrac{1}{2}\right)^2 \ge 0 \Rightarrow \left(3^x-\dfrac{1}{2}\right)^2 + \dfrac{3}{4} \ge 3/4$

$\Rightarrow\ f(x) \geq 3/4$

$\therefore$ Required range is $\left[\frac{3}{4}, \infty\right[$

10. (a) : $S = \frac{4}{19}+\frac{44}{19^2}+\frac{444}{19^3}+....\infty$ ...(i)

$\Rightarrow\ \frac{S}{19} = \frac{4}{19^2}+\frac{44}{19^3}+....\infty$ ...(ii)

Subtracting (ii) from (i), we get

$$\frac{18}{19}S = \frac{4}{19}+\frac{40}{19^2}+\frac{400}{19^3}+.....\infty$$

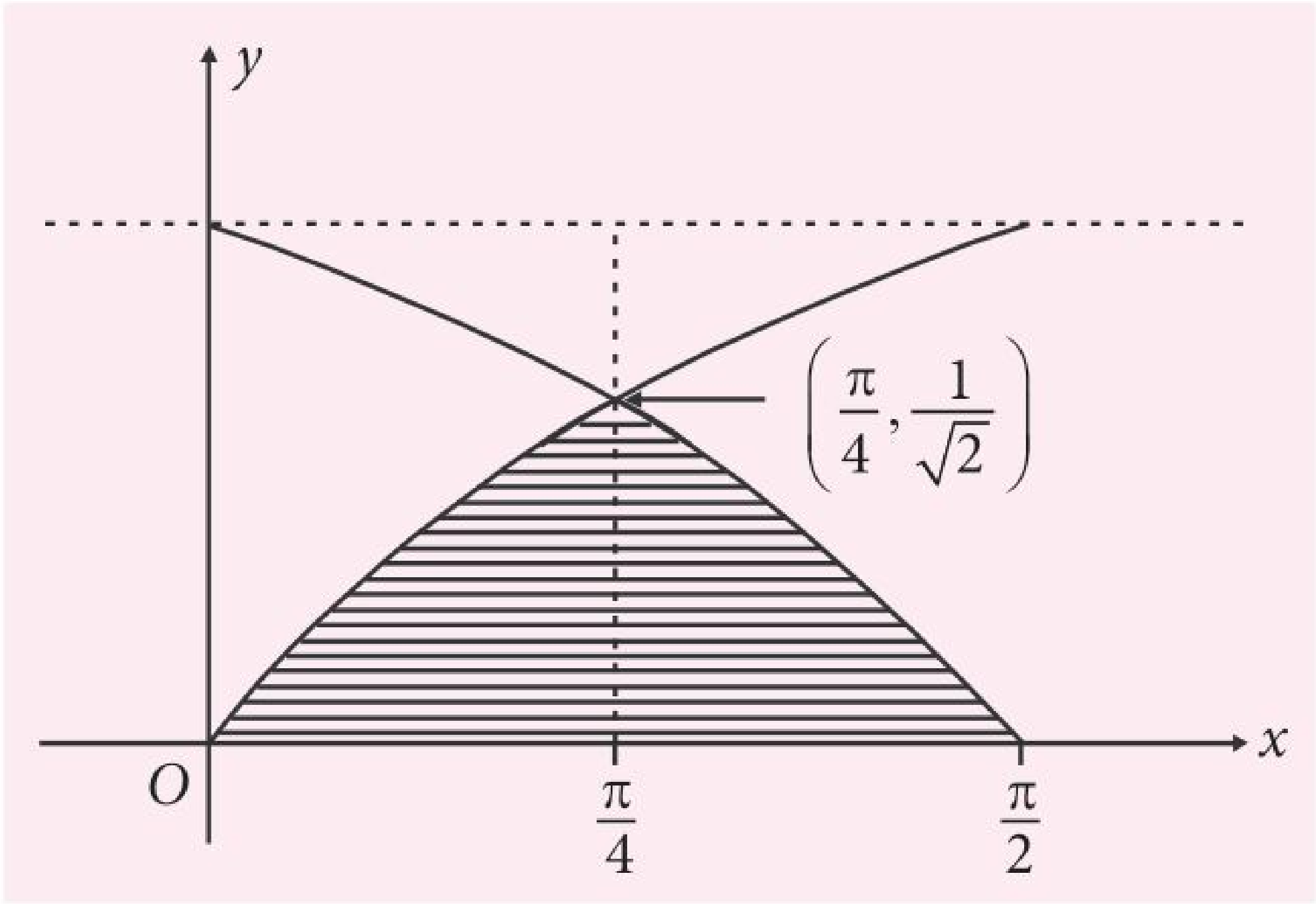
$$= \frac{4}{19}\left[1+\frac{10}{19}+\left(\frac{10}{19}\right)^2+.....\infty\right] = \frac{4}{19}\left[\frac{1}{1-\frac{10}{19}}\right] = \frac{4}{9}$$

$\Rightarrow\ S = \frac{76}{162} = \frac{38}{81}$

11. (b) : $f(x) = \min\{\sin x, \cos x\}$

Solving the equations $f(x) = \sin x$, $f(x) = \cos x$, we get

$\sin x = \cos x \Rightarrow x = \frac{\pi}{4}$



$\therefore\ f(x) = \min\{\sin x, \cos x\}$

$$= \begin{cases} \sin x & \forall \quad 0 \leq x \leq \frac{\pi}{4} \\ \cos x & \forall \quad \frac{\pi}{4} < x \leq \frac{\pi}{2} \end{cases}$$

$\therefore$ Required area $= \int_{0}^{\pi/4} \sin x\,dx + \int_{\pi/4}^{\pi/2} \cos x\,dx$

$$= (-\cos x)_0^{\pi/4} + (\sin x)_{\pi/4}^{\pi/2} = \left(-\frac{1}{\sqrt{2}}+1\right)+\left(1-\frac{1}{\sqrt{2}}\right)$$

$= 2-\sqrt{2} = \sqrt{2}(\sqrt{2}-1)$ sq. units

12. (a) : Let $\vec{a} = \hat{i}-2\hat{j}+3\hat{k}$, $\vec{b} = -2\hat{i}+3\hat{j}-4\hat{k}$

and $\vec{c} = \lambda\hat{i}-\hat{j}+2\hat{k}$ are coplanar.

$\therefore\ [\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ \lambda & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow\ 1(6 - 4) + 2(-4 + 4\lambda) + 3(2 - 3\lambda) = 0 \Rightarrow \lambda = 0$

13. (b) : Given, $iz^3 + z^2 - z + i = 0$

$\Rightarrow\ iz^2(z - i) - 1(z - i) = 0$

$\Rightarrow\ (z - i)(iz^2 - 1) = 0 \Rightarrow z = i$ or $iz^2 = 1$

$\Rightarrow\ |z| = 1$ or $iz^2 = 1$ …(i)

Now, $iz^2 = 1$

$\Rightarrow\ -i\bar{z}^2 = 1$ (By taking conjugate on both sides)...(ii)

On multiplying (i) and (ii), we get

$(z\bar{z})^2 = 1 \Rightarrow |z|^2 = 1 \Rightarrow |z| = 1$.

14. (b) : $\int \frac{\log\left(\frac{x}{e}\right)}{(\log x)^2}dx = \int \frac{\log x - 1}{(\log x)^2}dx$

$= \int e^t\left(\frac{1}{t}-\frac{1}{t^2}\right)dt$ (where $t = \log x$)

$= \frac{e^t}{t}+c = \frac{x}{\log x}+c$

15. (b) : Domain of $\sin^{-1}x$ is $[-1, 1]$

$\therefore$ Domain of $\sin^{-1}[x]$ is $\{x : -1 \leq [x] \leq 1\}$

$= \{x : [x] = -1, 0, 1\}$

But $[x] = \begin{cases} -1 & \forall \quad -1 \leq x < 0 \\ 0 & \forall \quad 0 \leq x < 1 \\ 1 & \forall \quad 1 \leq x < 2 \end{cases}$

$\therefore\ x \in [-1, 2)$ *i.e.* $-1 \leq x < 2$

16. (d) : Let $z = x + iy$ therefore the expression

$|z|^2 + |z - 3|^2 + |z - i|^2$

$= x^2 + y^2 + (x - 3)^2 + y^2 + x^2 + (y - 1)^2$

$$= 3\left\{(x-1)^2+\left(y-\frac{1}{3}\right)^2\right\}+\frac{20}{3}$$

$$= 3\left|(x-1)+i\left(y-\frac{1}{3}\right)\right|^2+\frac{20}{3} = 3\left|z-\left(1+\frac{i}{3}\right)\right|^2+\frac{20}{3}$$

$\therefore$ The value of $|z|^2 + |z - 3|^2 + |z - i|^2$ will be minimum when $z = 1+\frac{i}{3}$.

17. (c) : We have, $\lim\limits_{x\to1}\frac{(2x-3)\left(\sqrt{x}-1\right)}{2x^2+x-3} = \lim\limits_{x\to1}\frac{(2x-3)\left(\sqrt{x}-1\right)}{(2x+3)(x-1)}$

$$= \lim_{x\to1}\frac{(2x-3)\left(\sqrt{x}-1\right)}{(2x+3)\left(\sqrt{x}+1\right)\left(\sqrt{x}-1\right)}$$

$$= \lim_{x\to1}\frac{(2x-3)}{(2x+3)\left(\sqrt{x}+1\right)} = \frac{2\times1-3}{(2\times1+3)\left(\sqrt{1}+1\right)} = -\frac{1}{10}$$

18. (c) : Since $y = \frac{a^2}{x}$ $\therefore \frac{dy}{dx} = -\frac{a^2}{x^2}$

$\therefore$ At (x_1, y_1), $\frac{dy}{dx} = \frac{-a^2}{x_1^2}$

Thus tangent to the curve will be $y - y_1 = \frac{-a^2}{x_1^2}(x - x_1)$

$\Rightarrow \quad yx_1^2 - y_1x_1^2 = -a^2x + a^2x_1$

$\Rightarrow \quad a^2x + x_1^2y = x_1(x_1y_1 + a^2) = 2a^2x_1 \qquad (\because x_1y_1 = a^2)$

This meets the x-axis where $y = 0$

$\therefore \quad a^2x = 2a^2x_1 \Rightarrow x = 2x_1$

Thus, point on the x-axis is $(2x_1, 0)$

Again tangent meets the y-axis, where $x = 0$

$\therefore \quad x_1^2y = 2a^2x_1 \Rightarrow y = \frac{2a^2}{x_1}$

So point on the y-axis is $\left(0, \frac{2a^2}{x_1}\right)$

$\therefore \quad$ Required area $= \frac{1}{2}(2x_1)\left(\frac{2a^2}{x_1}\right) = 2a^2.$

19. (a) : Since, the system of equations is

$a_1x + b_1y + c_1z = 0$

$a_2x + b_2y + c_2z = 0$

$a_3x + b_3y + c_3z = 0$

Also $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

Therefore, this system has more than two solutions.

20. (a) : We have, $(a + bx)^{-2} = \frac{1}{4} - 3x$ +... (Given)

Also, $(a+bx)^{-2} = a^{-2}\left(1+\frac{b}{a}x\right)^{-2} = a^{-2}\left(1-\frac{2b}{a}x\right)$

[Neglecting x^2 and higher powers of x]

$= \frac{1}{a^2} - \frac{2bx}{a^3}$

$\therefore \quad \frac{1}{a^2} - \frac{2bx}{a^3} = \frac{1}{4} - 3x \Rightarrow \frac{1}{a^2} = \frac{1}{4}$ and $\frac{2b}{a^3} = 3$

$\Rightarrow \quad a = 2$ and $b = \frac{3}{2}a^3 = \frac{3}{2}\times(2)^3 = \frac{3}{2}\times 8 = 12$

21. (192) : Given digits are 3, 5, 6, 7 and 8

We need 4 digit numbers greater than 6000 and 5 digit numbers and will add them to get the result.

Now, 5 digit numbers $= {}^5P_5 = 5! = 120$

Now, for 4 digit numbers each greater than 6000, left extreme place can be filled in ${}^3C_1$ ways as this place has choices with 6, 7, 8 digits. Now the remaining three places can be filled by choices 4, 3 and 2 ways.

$\therefore \quad$ Required such numbers $= {}^3C_1 \times 4 \times 3 \times 2 = 72$

$\therefore \quad$ Required numbers greater than 6000
$= 120 + 72 = 192$

22 (0) : Let $I = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$ (Put $x = \cos 2\theta$)

$= \int -2\theta \sin 2\theta \, d\theta$

Using integration by parts, we get

$I = \theta \cos 2\theta - \frac{\sin 2\theta}{2} + c = \frac{x}{2}\cos^{-1}x - \frac{1}{2}\sqrt{1-x^2} + c$

$\Rightarrow \quad A = \frac{1}{2}, B = -\frac{1}{2}$

$\therefore \quad A + B = 0$

23. (3) : The given equation is $\frac{dy}{dx} = 3x \Rightarrow dy = 3x\,dx$

Integrating both sides, we get $y = \frac{3x^2}{2} + c$

Since, it passes through $(-1, -3)$, so $-3 = \frac{3}{2} + c$

$\therefore \quad c = -\frac{9}{2} \Rightarrow y = \frac{3x^2}{2} - \frac{9}{2} \Rightarrow 2y = 3(x^2 - 3)$

$\therefore \quad \lambda = 3$

24. (720) : Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$.

To find the number of bijective functions $f: A \to A$ such that $f(1) + f(2) + f(3) = 3$, the only possibility for this sum is $0 + 1 + 2 = 3$.

$\Rightarrow$ Elements 1, 2, 3 in the domain can be mapped with 0, 1, 2 only.

Therefore, number of bijective functions $= 3! \times 5!$
$= 6 \times 120 = 720.$

25. (0) : $a = 9, b = 40\lambda, c = 41\lambda$, area $= \Delta$

$16\Delta^2 = (81\lambda + 9)(81\lambda - 9)(9 + \lambda)(9 - \lambda)$

$= (81^2 \cdot \lambda^2 - 81)(81 - \lambda^2)$

$\Rightarrow \quad \frac{16\Delta^2}{81^2} = \left(\lambda^2 - \frac{1}{81}\right)(81 - \lambda^2) = -1 - \lambda^4 + \left(81 + \frac{1}{81}\right)\lambda^2$

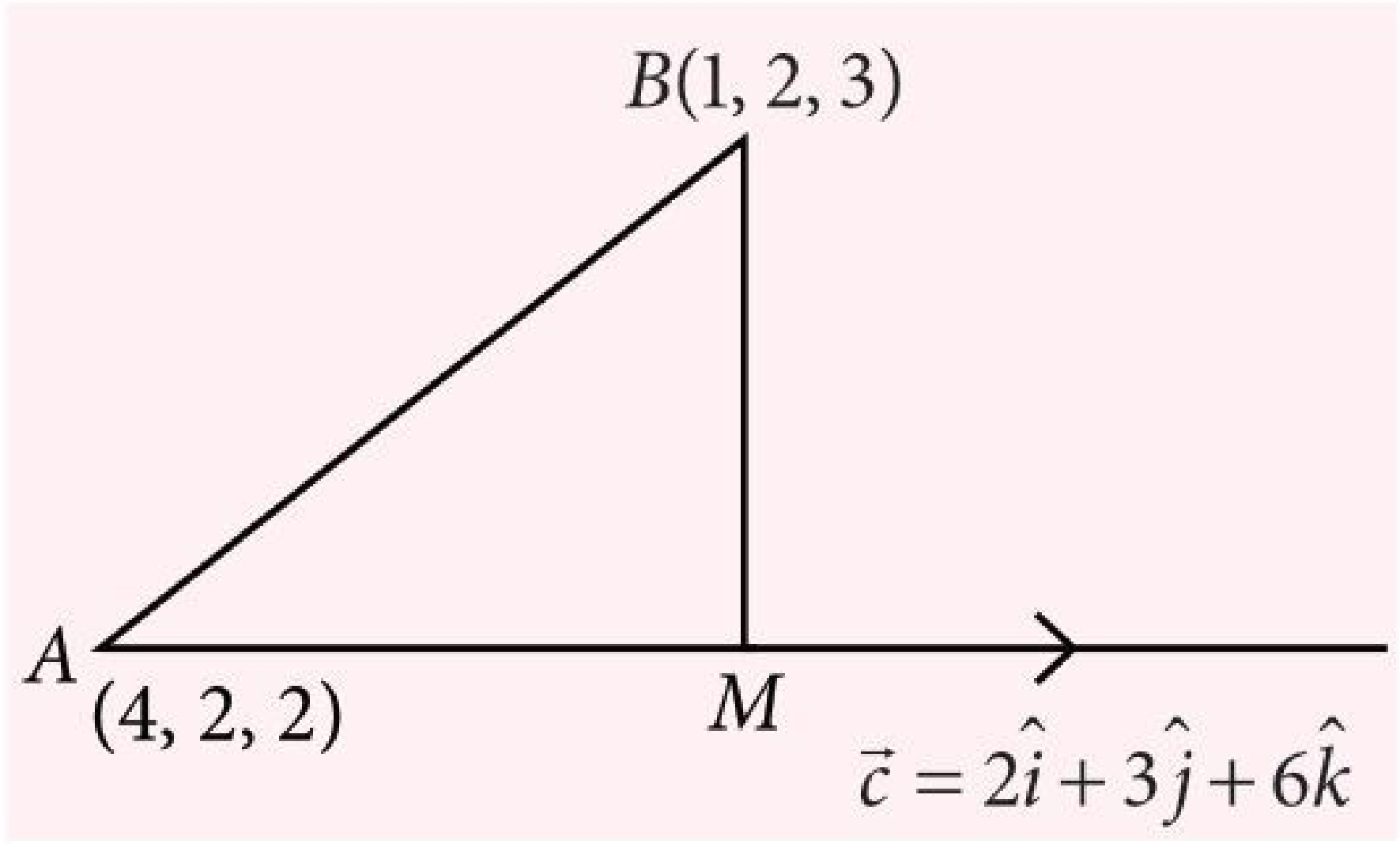
$= \frac{1}{4}\left(81 - \frac{1}{81}\right)^2 - \left(\lambda^2 - \frac{1}{2}\left(81 + \frac{1}{81}\right)\right)^2 \le \frac{1}{4}\left(81 - \frac{1}{81}\right)^2$

$\therefore \quad \frac{4\Delta}{81} \le \frac{1}{2}\left(81 - \frac{1}{81}\right) \Rightarrow \Delta \le \frac{81}{8}\left(81 - \frac{1}{81}\right) = \frac{81^2 - 1}{8} = 820$

26. (1111) : Clearly, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (\hat{i} + 2\hat{j} + 3\hat{k}) - (4\hat{i} + 2\hat{j} + 2\hat{k}) = -3\hat{i} + 0\hat{j} + \hat{k}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{9 + 0 + 1} = \sqrt{10}$



Now, AM = Projection of $\overrightarrow{AB}$ along $\vec{c} = \frac{\overrightarrow{AB} \cdot \vec{c}}{|\vec{c}|}$

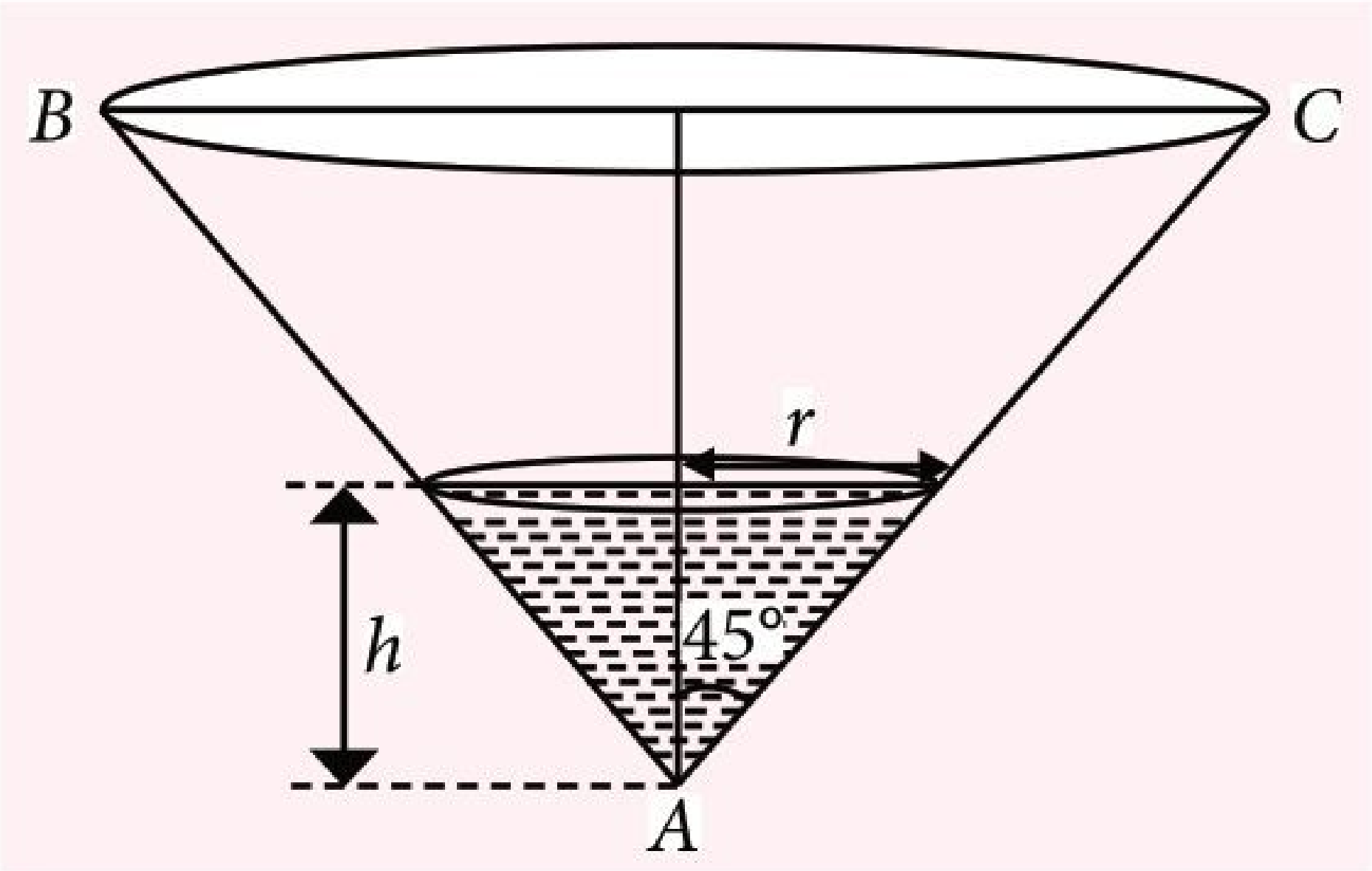
$$= \frac{(-3\hat{i}+0\hat{j}+\hat{k})\cdot(2\hat{i}+3\hat{j}+6\hat{k})}{\sqrt{(4+9+36)}} = \frac{-6+0+6}{7} = 0$$

$\therefore$ Perpendicular distance of B from

$AM = |\overrightarrow{AB}| = \sqrt{10} = \lambda$ (given)

$\therefore \quad \lambda^6 + \lambda^4 + \lambda^2 + 1 = 1000 + 100 + 10 + 1 = 1111$

27. (5) :



We have,

$$\frac{dV}{dt} = 2 \text{ or } \frac{d}{dt}\left(\frac{1}{3}\pi r^3\right) = 2 \qquad [\because r = h \text{ as } \theta = 45°]$$

$$\Rightarrow \pi r^2 \frac{dr}{dt} = 2 \Rightarrow \frac{dr}{dt} = \frac{2}{\pi r^2} \qquad \text{...(i)}$$

Now, perimeter $= 2\pi r = p$ (say)

$$\therefore \frac{d}{dt}(2\pi r) = 2\pi\frac{dr}{dt} = 2\pi\left(\frac{2}{\pi r^2}\right) = \frac{4}{r^2} \qquad \text{...(ii)}$$

[Using (i)]

When $h = 2$ m, $r = 2$ m

Hence, $\frac{dp}{dt} = \frac{4}{4} = 1$ m/s $= d$ m/s. Thus, $5d = 5$

28. (1) : $S(x, y) = x^2 + y^2 - x - y - 6 = 0$ ...(i)

has centre at $C \equiv \left(\frac{1}{2}, \frac{1}{2}\right)$

According to the required conditions, the given point $P(\alpha - 1, \alpha + 1)$ must lie inside the given circle.

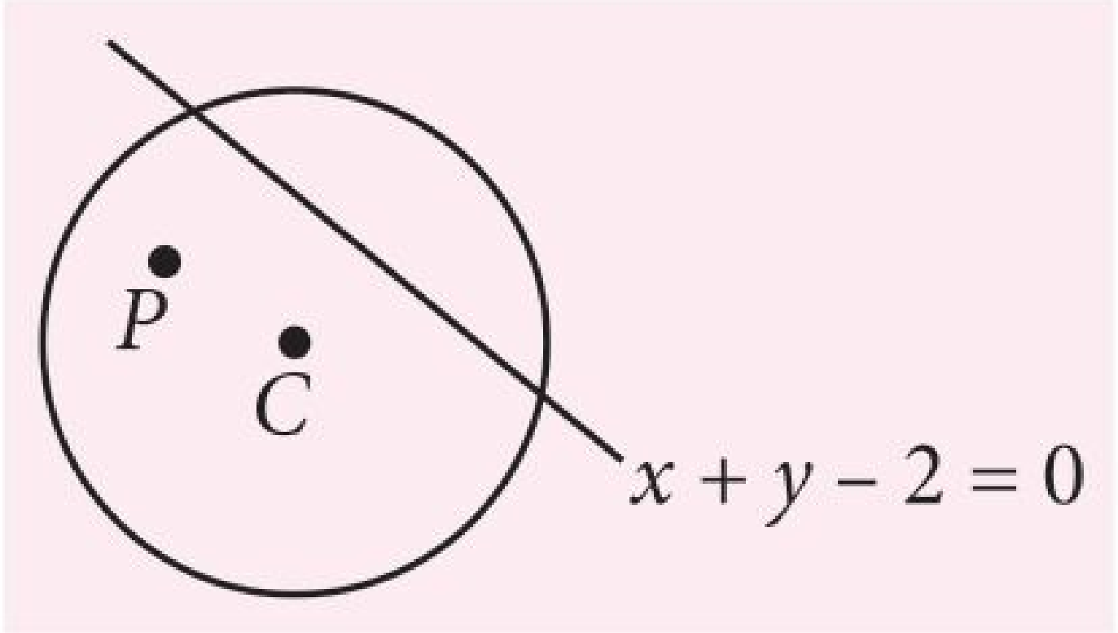
i.e. $S(\alpha - 1, \alpha + 1) < 0$

$\Rightarrow (\alpha - 1)^2 + (\alpha + 1)^2 - (\alpha - 1) - (\alpha + 1) - 6 < 0$

$\Rightarrow \alpha^2 - \alpha - 2 < 0 \quad \Rightarrow \quad (\alpha - 2)(\alpha + 1) < 0$

$\Rightarrow -1 < \alpha < 2$ ...(ii)

Also, P and C must lie on the same side of the line



$L(x, y) \equiv x + y - 2 = 0$ ...(iii)

i.e. $L(1/2, 1/2)$ and $L(\alpha - 1, \alpha + 1)$ must have the same sign.

Since $L\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2} + \frac{1}{2} - 2 < 0$

$\therefore \quad L(\alpha - 1, \alpha + 1) = (\alpha - 1) + (\alpha + 1) - 2 < 0$

i.e., $\alpha < 1$ ...(iv)

Inequalities (ii) and (iv) together give the permissible values of α as $-1 < \alpha < 1$.

29. (9) : If we have, $\begin{bmatrix} 2A+1 & -5 \\ -4 & A \end{bmatrix}^{-1} \begin{bmatrix} A-5 & B \\ 2A-2 & C \end{bmatrix} = \begin{bmatrix} 14 & S \\ E & F \end{bmatrix}$

$$\Rightarrow \frac{1}{(2A+1)A-20}\begin{bmatrix} A & 5 \\ 4 & 2A+1 \end{bmatrix}\begin{bmatrix} A-5 & B \\ 2A-2 & C \end{bmatrix} = \begin{bmatrix} 14 & S \\ E & F \end{bmatrix}$$

On comparing corresponding elements, we get

$$\frac{(A-5)A+10A-10}{2A^2+A-20} = 14$$

$\Rightarrow A^2 + 5A - 10 = 28A^2 + 14A - 280$

$\Rightarrow 27A^2 + 9A - 270 = 0$

$\Rightarrow A_1 + A_2 = \frac{-9}{27} \Rightarrow -27(A_1 + A_2) = 9$

30. (30) : For the first two draw, the balls taken out may be one of the following :

E_1 : White and White

E_2 : White and Black

E_3 : Black and White

E_4 : Black and Black

Then, $P(E_1) = P(W)\cdot P(W|W) = \frac{2}{4}\cdot\frac{1}{3} = \frac{1}{6}$

$P(E_2) = P(W)\cdot P(B|W) = \frac{2}{4}\cdot\frac{2}{3} = \frac{1}{3}$

$P(E_3) = P(B)\cdot P(W|B) = \frac{2}{4}\cdot\frac{2}{5} = \frac{1}{5}$

and $P(E_4) = P(B)\cdot P(B|B) = \frac{2}{4}\cdot\frac{3}{5} = \frac{3}{10}$

$\therefore \quad P(E_1) + P(E_2) + P(E_3) + P(E_4) = \frac{1}{6} + \frac{1}{3} + \frac{1}{5} + \frac{3}{10}$

$= \frac{10+20+12+18}{60} = 1$

Thus, events E_1, E_2, E_3 and E_4 are exhaustive. Also, these events are mutually exclusive.

Let B denote the event of drawing a black ball in the third attempt.

$\therefore \quad P(B|E_1) = 1, P(B|E_2) = \frac{3}{4},$

$P(B|E_3) = \frac{3}{4}$ and $P(B|E_4) = \frac{4}{6} = \frac{2}{3}$

$\therefore$ Required probability

P (black ball in third draw)

$= P(E_1)\cdot P(B|E_1) + P(E_2)\cdot P(B|E_2)$

$\qquad + P(E_3)\cdot P(B|E_3) + P(E_4)\cdot P(B|E_4)$

$= \frac{1}{6}\times 1 + \frac{1}{3}\times\frac{3}{4} + \frac{1}{5}\times\frac{3}{4} + \frac{3}{10}\times\frac{2}{3} = \frac{1}{6} + \frac{1}{4} + \frac{3}{20} + \frac{1}{5}$

$= \frac{10+15+9+12}{60} = \frac{46}{60} = \frac{23}{30} = \frac{23}{\lambda}$ (given)

$\Rightarrow \lambda = 30$

TARGET JEE

1. Between two junction stations A and B there are 12 intermediate stations. The number of ways in which a train can be made to stop at 4 of these stations so that no two of these halting stations are consecutive, is

(a) ${}^{8}C_4$ (b) ${}^{9}C_4$

(c) ${}^{12}C_4 - 4$ (d) none of these

2. If $\left|\dfrac{z-6}{z-8}\right| = 1$, then the value of $x \in \mathbf{R}$, where

$$z = x + i\begin{vmatrix} -3 & 2i & 2+i \\ -2i & 2 & 4-3i \\ 2-i & 4+3i & 7 \end{vmatrix}, \text{ is}$$

(a) 5 (b) 7

(c) 9 (d) cannot be determined

3. If $A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$, then $(A(\text{adj}A)A^{-1})A =$

(a) $2\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1/6 & -1/6 \\ 2/6 & 1/6 & 3/6 \\ 3/6 & 2/6 & 1/6 \end{bmatrix}$ (d) none of these

4. Let p be an odd prime number and T_p be the following set of 2×2 matrices.

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \dots, (p-1)\} \right\}$$

The number of matrices A in T_p such that A is either symmetric or skew symmetric or both and det(A) is divisible by p is

(a) $(p-1)^2$ (b) $2(p-1)$

(c) $(p-1)^2 + 1$ (d) $2p - 1$

5. If $A = \{\theta : 2\cos^2\theta + \sin\theta \le 2\}$ and

$B = \left\{\theta : \dfrac{\pi}{2} \le \theta \le \dfrac{3\pi}{2}\right\}$, then $A \cap B =$

(a) $\left\{\theta : \dfrac{\pi}{2} \le \theta \le \dfrac{5\pi}{6}\right\}$

(b) $\left\{\theta : \pi \le \theta \le \dfrac{3\pi}{2}\right\}$

(c) $\left\{\theta : \dfrac{\pi}{2} \le \theta \le \dfrac{5\pi}{6}\right\} \cup \left\{\theta : \pi \le \theta \le \dfrac{3\pi}{2}\right\}$

(d) none of these

6. The relation R defined on a set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a, b) : |a^2 - b^2| < 16\}$ is given by

(a) $R = \{(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)\}$

(b) $R = \{(2, 2), (3, 2), (4, 2), (2, 4)\}$

(c) $R = \{(3, 3), (4, 3), (5, 4), (3, 4)\}$

(d) none of these

7. If $f : A \to B$ given by $3^{f(x)} + 2^{-x} = 4$ is a bijection, then

(a) $A = \{x \in \mathbf{R} : -1 < x < \infty\}, B = \{x \in \mathbf{R} : 2 < x < 4\}$

(b) $A = \{x \in \mathbf{R} : -3 < x < \infty\}, B = \{x \in \mathbf{R} : 0 < x < 4\}$

(c) $A = \{x \in \mathbf{R} : -2 < x < \infty\}, B = \{x \in \mathbf{R} : 0 < x < 4\}$

(d) none of these

8. If 3 arithmetic means, 3 geometric means and 3 harmonic means are inserted between 1 and 5, then the cubic equation whose roots are first A.M., second G.M. and third H.M. between 1 and 5, is

(a) $x^3 - \left(\dfrac{9}{2} + \sqrt{5}\right)x^2 + \left(\dfrac{9\sqrt{5}}{2} + 5\right)x - 5\sqrt{5} = 0$

(b) $x^3 + \left(\dfrac{9}{2} + \sqrt{5}\right)x^2 - \left(\dfrac{9\sqrt{5}}{2} + 5\right)x - 5\sqrt{5} = 0$

(c) $x^3+\left(\frac{9}{2}-\sqrt{5}\right)x^2-\left(\frac{9\sqrt{5}}{2}-5\right)x+5\sqrt{5}=0$

(d) none of these

9. A circle circumscribing an equilateral triangle with centroid at (0, 0) of side 'a' is drawn and a square is drawn whose four sides touch the circle. The equation of the circle circumscribing the square is

(a) $x^2 + y^2 = 2a^2$ (b) $3x^2 + 3y^2 = 2a^2$
(c) $5x^2 + 5y^2 = 3a^2$ (d) none of these

10. Let X and Y be two events such that $P(X/Y)=\frac{1}{2}, P(Y/X)=\frac{1}{3}$ and $P(X\cap Y)=\frac{1}{6}$. Which of the following is incorrect?

(a) $P(X\cup Y)=\frac{2}{3}$
(b) X and Y are independent events
(c) $P(\bar{X}\cap Y)=\frac{1}{6}$
(d) X and Y are not independent events

11. If 1 lies between the roots of the equation $y^2 - my + 1 = 0$ and $[x]$ denotes the greatest integer less than or equal to x then the value of $\left[\left(\frac{4|x|}{|x|^2+16}\right)^m\right]$ is

(a) 0 (b) 1
(c) 2 (d) undefined

12. If the sum of the coefficients in the expansion of $(b + c)^{20}$ $[1 + (a - 2)x]^{20}$ is equal to square of the sum of the coefficients in the expansion of $[2bcx - (b + c)y]^{10}$, where a, b, c are positive constants, then

(a) $a\geq\sqrt{bc}$ (b) $\frac{b+c}{2}\geq a$
(c) $\frac{1}{c},\frac{1}{a},\frac{1}{b}$ are in G.P. (d) none of these

13. The value of sec40° + sec80° + sec160° will be

(a) 4 (b) −4
(c) 6 (d) 8

14. If in a ΔABC, $\sin A$, $\sin B$ and $\sin C$ are in A.P., then

(a) the altitudes are in A.P.
(b) the altitudes are in H.P.
(c) the medians are in G.P.
(d) the medians are in A.P.

15. The value of $\tan\{\sin^{-1}(\cos(\sin^{-1}x))\}$ $\tan\{\cos^{-1}(\sin(\cos^{-1}x))\}$, where $0<x<\frac{\pi}{2}$ is equal to

(a) 0 (b) 1
(c) −1 (d) none of these

16. Let S and S' be two foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If a circle described on SS' as diameter intersects the ellipse in real and distinct points, then the eccentricity e of the ellipse satisfies

(a) $e=1/\sqrt{2}$ (b) $e\in(1/\sqrt{2},1)$
(c) $e\in(0,1/\sqrt{2})$ (d) none of these

17. A family of lines given by $(1 + 2\lambda)x + (1 - \lambda)y + \lambda = 0$, λ being the parameter. The line belonging to this family at the maximum distance from the point (1, 4), is

(a) $4x - y + 1 = 0$ (b) $33x + 12y + 7 = 0$
(c) $12x + 33y = 7$ (d) none of these

18. The value of 'a' so that the volume of the parallelepiped formed by $\hat{i}+a\hat{j}+\hat{k}$, $\hat{j}+a\hat{k}$, $a\hat{i}+\hat{k}$ becomes minimum is

(a) 1/3 (b) 3 (c) $1/\sqrt{3}$ (d) $\sqrt{3}$

19. $\int\sin(101x)\sin^{99}x\,dx$ equals

(a) $\frac{1}{100}\sin(100x)(\sin x)^{100}+C$
(b) $\frac{1}{100}\cos(100x)(\sin x)^{100}+C$
(c) $\frac{1}{100}\cos(100x)(\cos x)^{100}+C$
(d) $\frac{1}{100}\sin(100x)(\sin x)^{101}+C$

20. Tangent is drawn at any point P of the curve which passes through (1, 1) cutting x-axis and y-axis at A and B respectively. If $AP : PB = 3 : 1$, then

(a) the differential equation of the curve is $3x\frac{dy}{dx}+y=0$ and the curve passes through (1/8, 2)
(b) the differential equation of the curve is $3x\frac{dy}{dx}-y=0$ and the curve passes through (1/8, −2)
(c) the curve is passing through (−1/8, 2)
(d) the normal at (1, 1) is $x + 3y = 4$

21. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{a}\cdot\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$, then $\vec{b}$ is

(a) $\hat{i}-\hat{j}+\hat{k}$ (b) $2\hat{j}-\hat{k}$
(c) $\hat{i}$ (d) $2\hat{i}$

22. If a curve is represented parametrically by the equations : $x = 4t^3 + 3$ and $y = 4 + 3t^4$ and $\dfrac{d^2x/dy^2}{(dx/dy)^n}$ is constant, then the value of n is

(a) 3 (b) 4

(c) 5 (d) 6

23. If a particle moving along a line follows the law $s=\sqrt{1+t}$, then the acceleration is proportional to

(a) square of the velocity

(b) cube of the displacement

(c) cube of the velocity

(d) square of the displacement

24. The lines $\dfrac{x-2}{1}=\dfrac{y-3}{1}=\dfrac{z-4}{-k}$ and $\dfrac{x-1}{k}=\dfrac{y-4}{2}=\dfrac{z-5}{1}$ are coplanar, iff

(a) $k = 3$ or -3 (b) $k = 0$ or -1

(c) $k = 1$ or -1 (d) $k = 0$ or -3

25. Let $\vec{A}$ be a vector parallel to line of intersection of planes P_1 and P_2 through the origin. P_1 is parallel to the vectors $2\hat{j}+3\hat{k}$ and $4\hat{j}-3\hat{k}$, P_2 is parallel to $\hat{j}-\hat{k}$ and $3\hat{i}+3\hat{j}$, then the angle between $\vec{A}$ and $2\hat{i}+\hat{j}-2\hat{k}$ is

(a) $\dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$ (b) $\dfrac{\pi}{2}$ or $\dfrac{3\pi}{2}$

(c) $\dfrac{\pi}{6}$ or $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$

26. Area bounded by $|x - 1| \le 2$ and $x^2 - y^2 = 1$, is

(a) $6\sqrt{2}+\frac{1}{2}\log|3+2\sqrt{2}|$ (b) $6\sqrt{2}+\frac{1}{2}\log|3-2\sqrt{2}|$

(c) $6\sqrt{2}-\log|3+2\sqrt{2}|$ (d) none of these

27. The mean deviation and standard deviation about actual mean of the series $a, a + d, a + 2d, \ldots, a + 2nd$ are respectively

(a) $\dfrac{n(n+1)d}{2n+1}, \sqrt{\dfrac{n(n-1)}{3}}\cdot d$

(b) $\dfrac{n(n-1)}{3}, \dfrac{n(n+1)}{2n}\cdot d$

(c) $\dfrac{n(n+1)d}{2n+1}, \sqrt{\dfrac{n(n+1)}{3}}\cdot d$

(d) $\dfrac{n(n-1)d}{2n-1}, \sqrt{\dfrac{n(n-1)}{3}}\cdot d$

28. Let f be a function defined by $f(x)=\int_1^x t(t^2-3t+2)dt$, $1 \le x \le 3$. Then the range of $f(x)$ is

(a) $[0, 2]$ (b) $[-1/4, 4]$

(c) $[-1/4, 2]$ (d) none of these

29. The set of values of b for which $f(x)$ has greatest value at $x = 1$, where $f(x)=\begin{cases} x^3-x^2+10x-7, & x<1 \\ -2x+\log_2(b^2-4), & x>1 \end{cases}$ is

(a) $(-6, -2)$ (b) $(2, 6)$

(c) $(-6, -2) \cup (2, 6)$ (d) $(-6, 6)$

30. Let $f : \mathbf{R} \to \mathbf{R}$ be a function such that $f(x) = ax + 3\sin x + 4\cos x$. Then $f(x)$, is invertible if

(a) $a \in (-5, 5)$ (b) $a \in (-\infty, -5)$

(c) $a \in (5, \infty)$ (d) Both (b) and (c)

SOLUTIONS

1. (b) : Let x_1 be the number of stations before the first halting station, x_2 between first and second, x_3 between second and third, x_4 between third and fourth and x_5 on the right of the fourth station.

Then, $x_1 \ge 0$, $x_5 \ge 0$, $x_2, x_3, x_4 \ge 1$ such that

$$x_1 + x_2 + x_3 + x_4 + x_5 = 8 \quad \ldots(i)$$

The total number of ways is the number of solutions of the above equation.

Let $y_2 = x_2 - 1$, $y_3 = x_3 - 1$, $y_4 = x_4 - 1$. Then (i) becomes

$$x_1 + y_2 + y_3 + y_4 + x_5 = 5 \quad \ldots(ii)$$

where $y_2, y_3, y_4 \ge 0$

The number of solutions of (ii) are

$$^{5+5-1}C_{5-1} = {}^9C_4$$

2. (b) : We have $\left|\dfrac{z-6}{z-8}\right|=1$

$\Rightarrow$ $|z - 6| = |z - 8|$

$\Rightarrow$ z lies on the perpendicular bisector of the segment joining (6, 0) and (8, 0) *i.e.* on $x = 7$.

$\Rightarrow$ $z = 7 + iy$, where $y \in \mathbf{R}$ ...(i)

Now, $z=x+i\begin{vmatrix} -3 & 2i & 2+i \\ -2i & 2 & 4-3i \\ 2-i & 4+3i & 7 \end{vmatrix}=x+35i$ ...(ii)

From (i) and (ii), we get $x = 7$

3. (a) : We have, $|A|=\begin{vmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{vmatrix}=0+7-1=6$

$\therefore$ $(A(\text{adj}A)A^{-1})A = (A(\text{adj}A))(A^{-1}A) = (|A|I)I$

$$\Rightarrow\ (A(\text{adj}A)A^{-1})A = |A|I = \begin{bmatrix}6&0&0\\0&6&0\\0&0&6\end{bmatrix} = 2\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$$

4. (d) : Case-I : When A is both symmetric as well as skew-symmetric.

In this case, we have $A = 0$ (Null matrix)

$\therefore\ \ |A| = 0 \ \ \Rightarrow\ \ |A|$ is divisible by p.

Case-II : When A is skew-symmetric matrix.

In this case, we have $a = 0$ and $c = -b$

$$\therefore\ \ |A| = \begin{vmatrix}0&b\\-b&0\end{vmatrix} = b^2$$

Now, $p/|A| \ \ \Rightarrow\ \ p/b^2 \ \ \Rightarrow\ \ b = 0$

[$\because\ b \in \{0, 1, 2, 3, ..., (p-1)\}$]

$$\therefore\ \ A = \begin{bmatrix}0&0\\0&0\end{bmatrix}$$

Case-III : When A is symmetric matrix

In this case, we have $c = b$

$$\therefore\ \ A = \begin{bmatrix}a&b\\b&a\end{bmatrix} \ \ \Rightarrow\ \ |A| = a^2 - b^2 = (a-b)(a+b)$$

$\Rightarrow\ \ p/(a-b)$ or $p/(a+b)$ [$\because\ p$ is prime]

When $p/(a-b) \ \Rightarrow\ a = b$. So, there are p choices for matrix A including $A = 0$.

When $p/(a+b) \ \Rightarrow\ a = r,\ b = p - r$, where $r = 0, 1, 2, ..., p - 1$.

So, there are p choices for matrix A including $A = 0$.

Hence, there are $(2p - 1)$ choices.

5. (c) : We have, $2\cos^2\theta + \sin\theta \le 2$

$\Rightarrow\ \ 2(1 - \sin^2\theta) + \sin\theta \le 2 \ \Rightarrow\ 2\sin^2\theta - \sin\theta \ge 0$

$\Rightarrow\ \ \sin\theta(2\sin\theta - 1) \ge 0$

Now, following two cases arises :

Case I: When $\sin\theta \ge 0$ and $(2\sin\theta - 1) \ge 0$

$$\Rightarrow\ \sin\theta \ge 0 \text{ and } \sin\theta \ge \frac{1}{2} \ \ \Rightarrow\ \ \frac{\pi}{6} \le \theta \le \frac{5\pi}{6}$$

$$\therefore\ \ A \cap B = \left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6}\right\} \left[\because B = \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\right]$$

Case II : When $\sin\theta \le 0$ and $(2\sin\theta - 1) \le 0$

$$\Rightarrow\ \ \sin\theta \le 0 \text{ and } \sin\theta \le \frac{1}{2}$$

$\Rightarrow\ \ \sin\theta \le 0 \ \ \Rightarrow\ \ \pi \le \theta \le 2\pi$

$$\therefore\ \ A \cap B = \left\{\theta : \pi \le \theta \le \frac{3\pi}{2}\right\} \left[\because B = \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\right]$$

$$\text{Thus, } A \cap B = \left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6}\right\} \cup \left\{\theta : \pi \le \theta \le \frac{3\pi}{2}\right\}$$

6. (d) : We have, $(a, b) \in R \Leftrightarrow |a^2 - b^2| < 16$

When $a = 1 \ \Rightarrow\ |1 - b^2| < 16 \ \Rightarrow\ |b^2 - 1| < 16$

$\Rightarrow\ \ -15 < b^2 < 17 \ \Rightarrow\ 0 < b^2 < 17 \ \Rightarrow\ b = 1, 2, 3, 4$

When $a = 2 \ \Rightarrow\ |4 - b^2| < 16 \ \Rightarrow\ |b^2 - 4| < 16$

$\Rightarrow\ \ -12 < b^2 < 20 \ \Rightarrow\ 0 < b^2 < 20 \ \Rightarrow\ b = 1, 2, 3, 4$

When $a = 3 \ \Rightarrow\ |9 - b^2| < 16 \ \Rightarrow\ |b^2 - 9| < 16$

$\Rightarrow\ \ -7 < b^2 < 25 \ \Rightarrow\ 0 < b^2 < 25 \ \Rightarrow\ b = 1, 2, 3, 4$

When $a = 4 \ \Rightarrow\ |16 - b^2| < 16 \ \Rightarrow\ |b^2 - 16| < 16$

$\Rightarrow\ \ 0 < b^2 < 32 \ \Rightarrow\ b = 1, 2, 3, 4, 5$

When $a = 5 \ \Rightarrow\ |25 - b^2| < 16 \ \Rightarrow\ |b^2 - 25| < 16$

$\Rightarrow\ \ 9 < b^2 < 41 \ \Rightarrow\ b = 4, 5.$

$$\therefore\ \ R = \begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),\\(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4),\\(4,5),(5,4),(5,5)\end{Bmatrix}$$

7. (d) : We have, $3^{f(x)} + 2^{-x} = 4$

$\Rightarrow\ \ 3^{f(x)} = 4 - 2^{-x} \ \ \Rightarrow\ \ f(x) = \log_3(4 - 2^{-x})$

Clearly, $f(x)$ will be defined, if $4 - 2^{-x} > 0$

$\Rightarrow\ \ 2^{-x} < 4 \ \ \Rightarrow\ 2^x > 2^{-2} \ \ \Rightarrow\ \ x > -2$

$\Rightarrow\ \ A = \{x \in \mathbf{R} : x > -2\}$

Now, $f(x) = \log_3(4 - 2^{-x})$

$$\Rightarrow\ \ f'(x) = \frac{2^{-x}\log 2}{(4 - 2^{-x})\log 3} > 0\ \ \forall\, x > -2$$

$\Rightarrow\ \ f(x)$ is increasing on $(-2, \infty)$

$\Rightarrow\ \ $ Range of $f(x) = \{y : f(-2) < y < f(\infty)\}$

$\Rightarrow\ \ B = \{y : -\infty < y < \log_3 4\} = \{x : -\infty < x < \log_3 4\}$

8. (a) : Let a = First A.M. between 1 and 5

$$\Rightarrow\ \ a = 1 + \frac{5-1}{3+1} = 2$$

b = Second G.M. between 1 and 5

$$\Rightarrow\ \ b = 1\left(\frac{5}{1}\right)^{2/4} = \sqrt{5}$$

c = Third H.M. between 1 and 5

$$\Rightarrow\ \ c = \frac{1}{1 + 3 \times \left(\frac{1}{5} - 1\right) \times \frac{1}{4}} = \frac{5}{2}$$

$$\therefore\ \ a + b + c = 2 + \sqrt{5} + \frac{5}{2},\ ab + bc + ca = 2\sqrt{5} + \frac{5\sqrt{5}}{2} + 5$$

and $abc = 5\sqrt{5}$

The cubic equation having a, b, c as its roots is

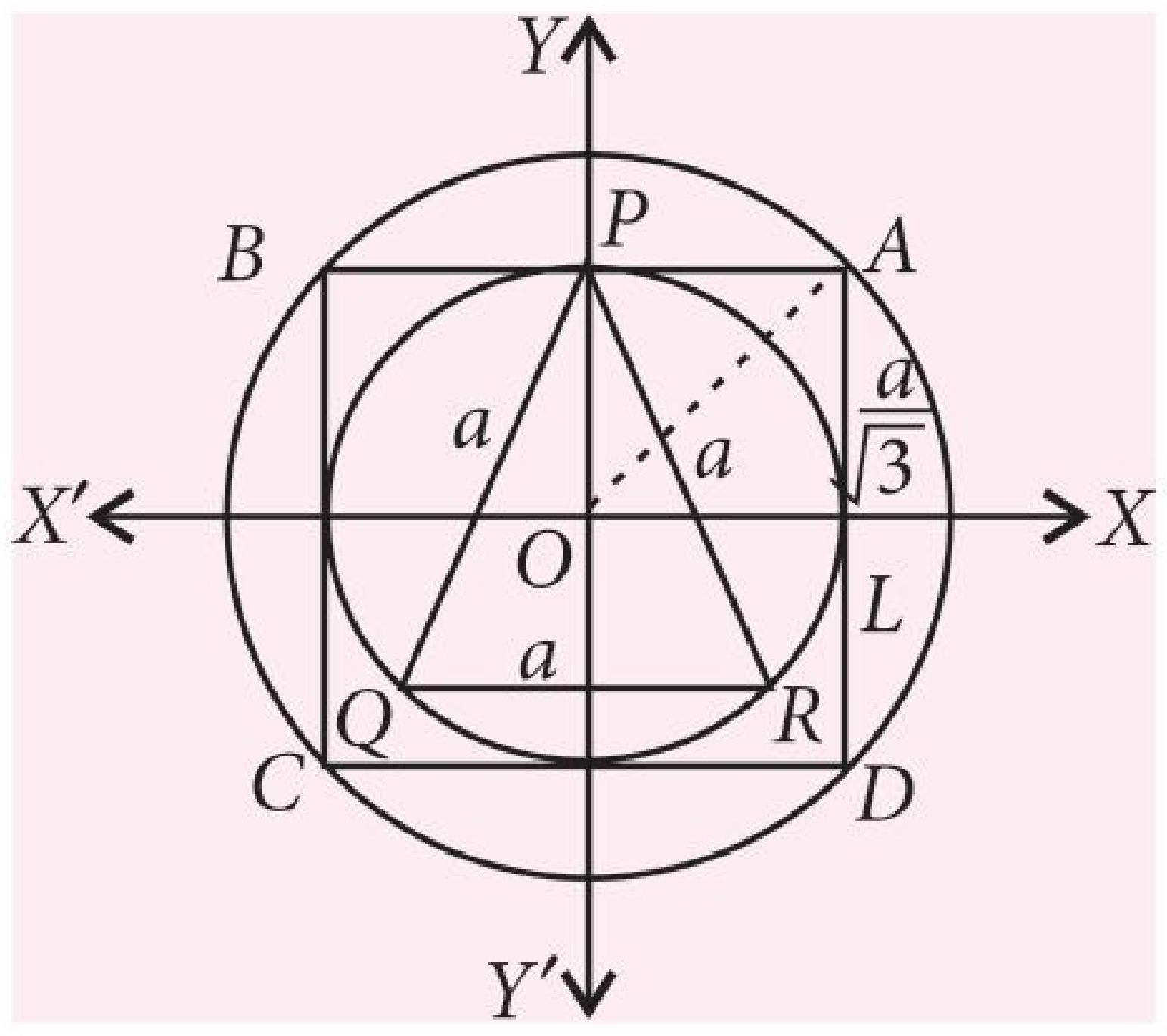
$$x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc = 0$$

$$\Rightarrow\ \ x^3 - \left(\frac{9}{2} + \sqrt{5}\right)x^2 + \left(\frac{9\sqrt{5}}{2} + 5\right)x - 5\sqrt{5} = 0.$$

9. (b) : Clearly, centre of the circumcircle is at origin and radius is two-third of the altitude $\frac{\sqrt{3}}{2}a$.

So, the equation of the circumcircle is

$$x^2+y^2=\left(\frac{a}{\sqrt{3}}\right)^2$$



Clearly the radius of the circumcircle of the square $ABCD$

$$=OA=\sqrt{OL^2+AL^2}$$

$$=\sqrt{\frac{a^2}{3}+\frac{a^2}{3}}=\sqrt{\frac{2}{3}}a$$

So, the equation of the circumcircle is

$$x^2+y^2=\frac{2}{3}a^2 \Rightarrow 3(x^2+y^2)=2a^2$$

10. (d) : Given, $P(X/Y)=\frac{1}{2}$, $P(Y/X)=\frac{1}{3}$ and $P(X\cap Y)=\frac{1}{6}$

$$\therefore\quad P(X)=\frac{P(X\cap Y)}{P(Y/X)}=\frac{\frac{1}{6}}{\frac{1}{3}}=\frac{1}{2}$$

Also, $P(Y)=\frac{P(X\cap Y)}{P(X/Y)}=\frac{\frac{1}{6}}{\frac{1}{2}}=\frac{1}{3}$

Clearly, $P(X\cap Y)=P(X)P(Y)$. So X and Y are independent events.

$$\therefore\quad P(X\cup Y)=1-P(\bar{X})P(\bar{Y})=1-\frac{1}{2}\times\frac{2}{3}=\frac{2}{3}.$$

11. (a) : Let $f(y)=y^2-my+1$. Since 1 lies between the roots of the equation $y^2-my+1=0$

$\therefore\quad m^2-4>0$ and $1-m+1<0$

$[\because D>0 \text{ and } f(1)<0]$

$\Rightarrow\quad (m<-2 \text{ or } m>2)$ and $m>2 \Rightarrow m>2$

Now, using A.M. – G.M. inequality, we get

$$\frac{|x|^2+16}{2}\geq 4|x| \Rightarrow 0\leq\frac{4|x|}{|x|^2+16}\leq\frac{1}{2}$$

$$\Rightarrow 0<\left(\frac{4|x|}{|x|^2+16}\right)^m<1 \Rightarrow \left[\left(\frac{4|x|}{|x|^2+16}\right)^m\right]=0.$$

12. (b) : The sum of the coefficients in the expansion of $(b+c)^{20}[1+(a-2)x]^{20}$ is given by

$S_1=(b+c)^{20}(a-1)^{20}$ [Putting $x=1$]

The sum of the coefficients in the expansion of $[2bcx-(b+c)y]^{10}$ is given by

$S_2=[2bc-(b+c)]^{10}$ [Putting $x=y=1$]

It is given that $S_1=S_2^2$

$\Rightarrow\quad (b+c)^{20}(a-1)^{20}=[2bc-(b+c)]^{20}$

$\Rightarrow\quad (b+c)(a-1)=[2bc-(b+c)]$

$$\Rightarrow (a-1)=\frac{2bc}{b+c}-1 \Rightarrow a=\frac{2bc}{b+c}$$

$\Rightarrow\quad a$ is the H.M. of b and c.

But, $\frac{b+c}{2}$ is the A.M. of b and c.

Using A.M. ≥ G.M. ≥ H.M., we get $\frac{b+c}{2}\geq a$.

13. (c): We find that $\cos40°+\cos80°+\cos160°$

$=2\cos60°\cos20°-\cos20°=0$ ...(i)

Now, $\cos40°\cos80°+\cos80°\cos160°+\cos160°\cos40°$

$$=\frac{1}{2}(2\cos40°\cos80°+2\cos80°\cos160°+2\cos160°\cos40°)$$

$$=\frac{1}{2}(\cos120°+\cos40°+\cos240°+\cos80°+\cos200°+\cos120°)$$

$$=\frac{1}{2}\left(-\frac{1}{2}+\cos40°-\frac{1}{2}+\cos80°+\cos200°-\frac{1}{2}\right)$$

$$=\frac{1}{2}\left(-\frac{3}{2}+2\cos60°\cos20°-\cos20°\right)=-\frac{3}{4} \quad \text{[Using (i)]}$$

Also, $\cos40°\cos80°\cos160°$

$$=\frac{\sin(2^3\times40°)}{2^3\sin40°}=\frac{\sin320°}{8(\sin40°)}=-\frac{1}{8}$$

$$\left[\because \cos\alpha\cos2\alpha\cos4\alpha\ldots\cos(2^{n-1}\alpha)=\frac{\sin2^n\alpha}{2^n\sin\alpha} \text{ for all } n\in N\right]$$

So, the equation having cos40°, cos80° and cos160° as its roots is given by

$$x^3-x^2\times0+x\times\left(-\frac{3}{4}\right)-\left(-\frac{1}{8}\right)=0 \text{ or } 8x^3-6x+1=0$$

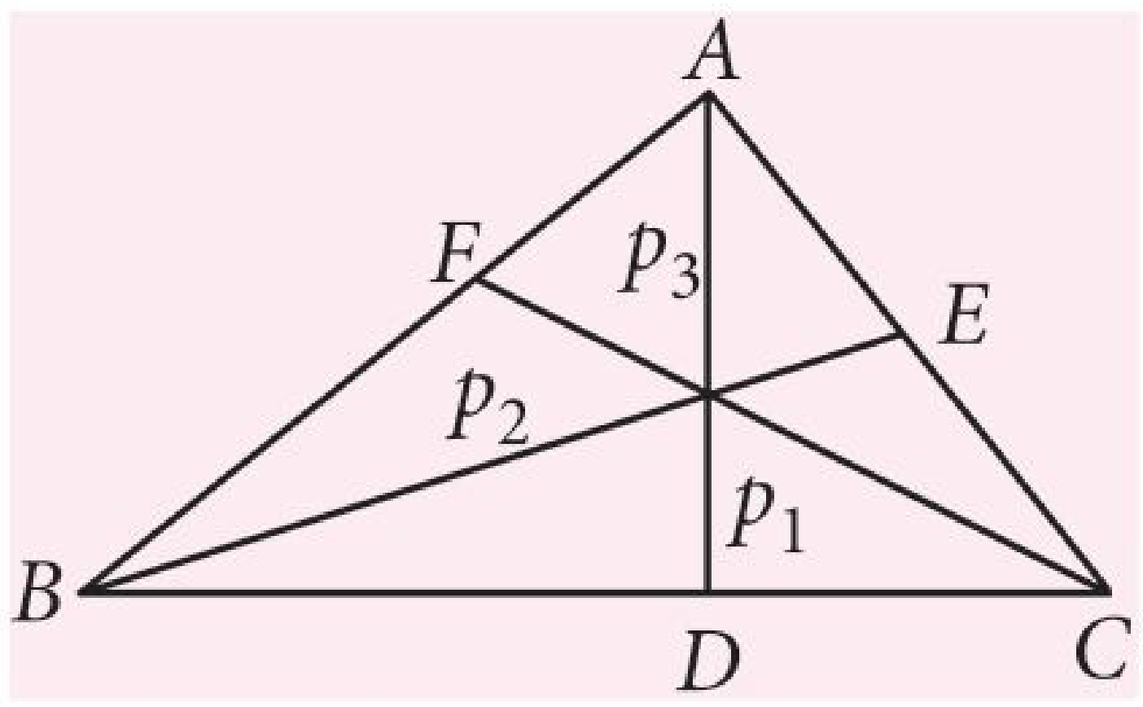
The equation having sec40°, sec80° and sec160° as its roots is $x^3-6x^2+8=0$

$\therefore\quad \sec40°+\sec80°+\sec160°=6$

14. (b) : Let p_1, p_2, p_3 be the altitudes of ΔABC

Then, $\Delta=\frac{1}{2}ap_1=\frac{1}{2}bp_2=\frac{1}{2}cp_3$

$$\Rightarrow a=\frac{2\Delta}{p_1}, b=\frac{2\Delta}{p_2}, c=\frac{2\Delta}{p_3}$$



Now, $\sin A$, $\sin B$, $\sin C$, are in A.P.

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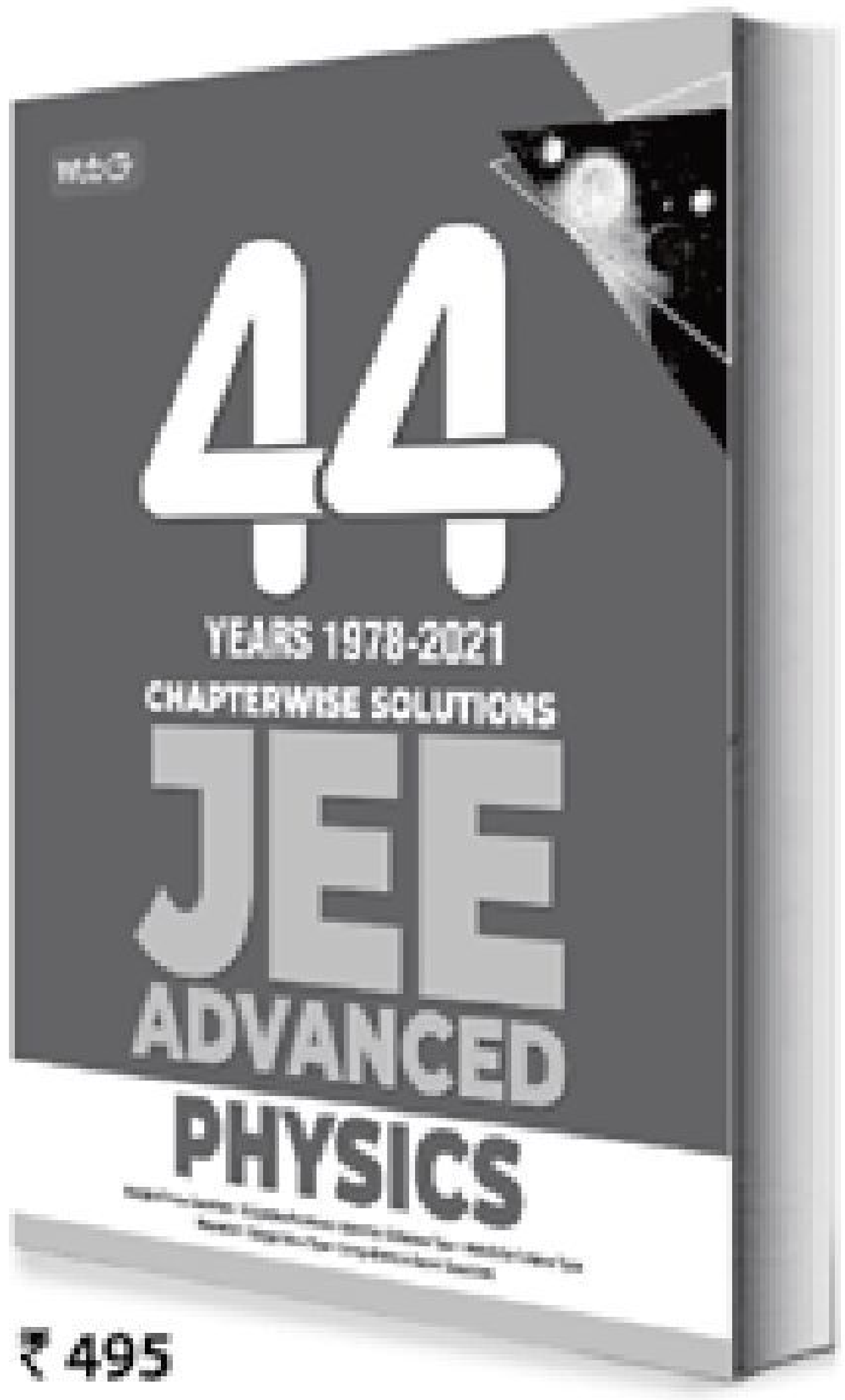
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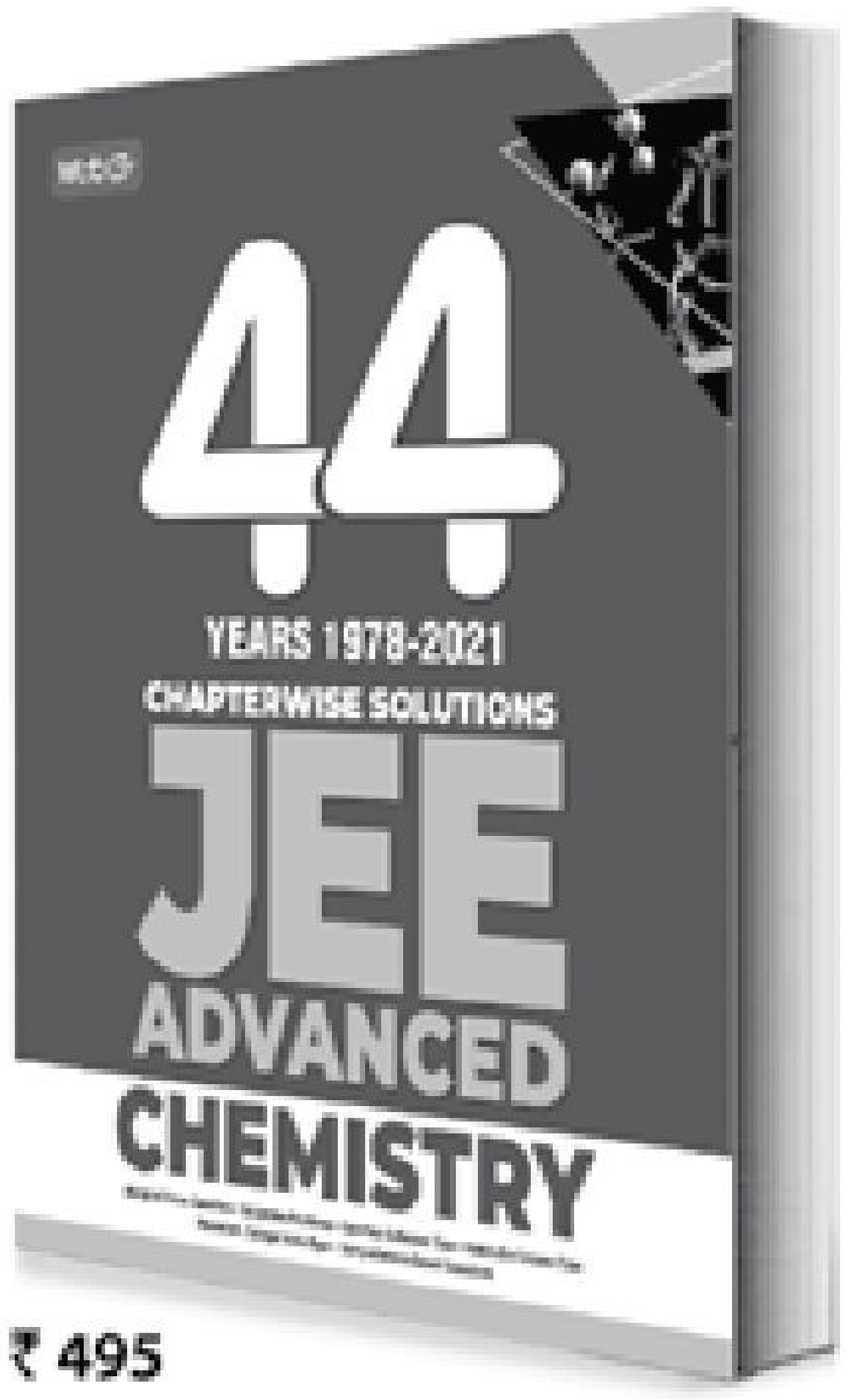
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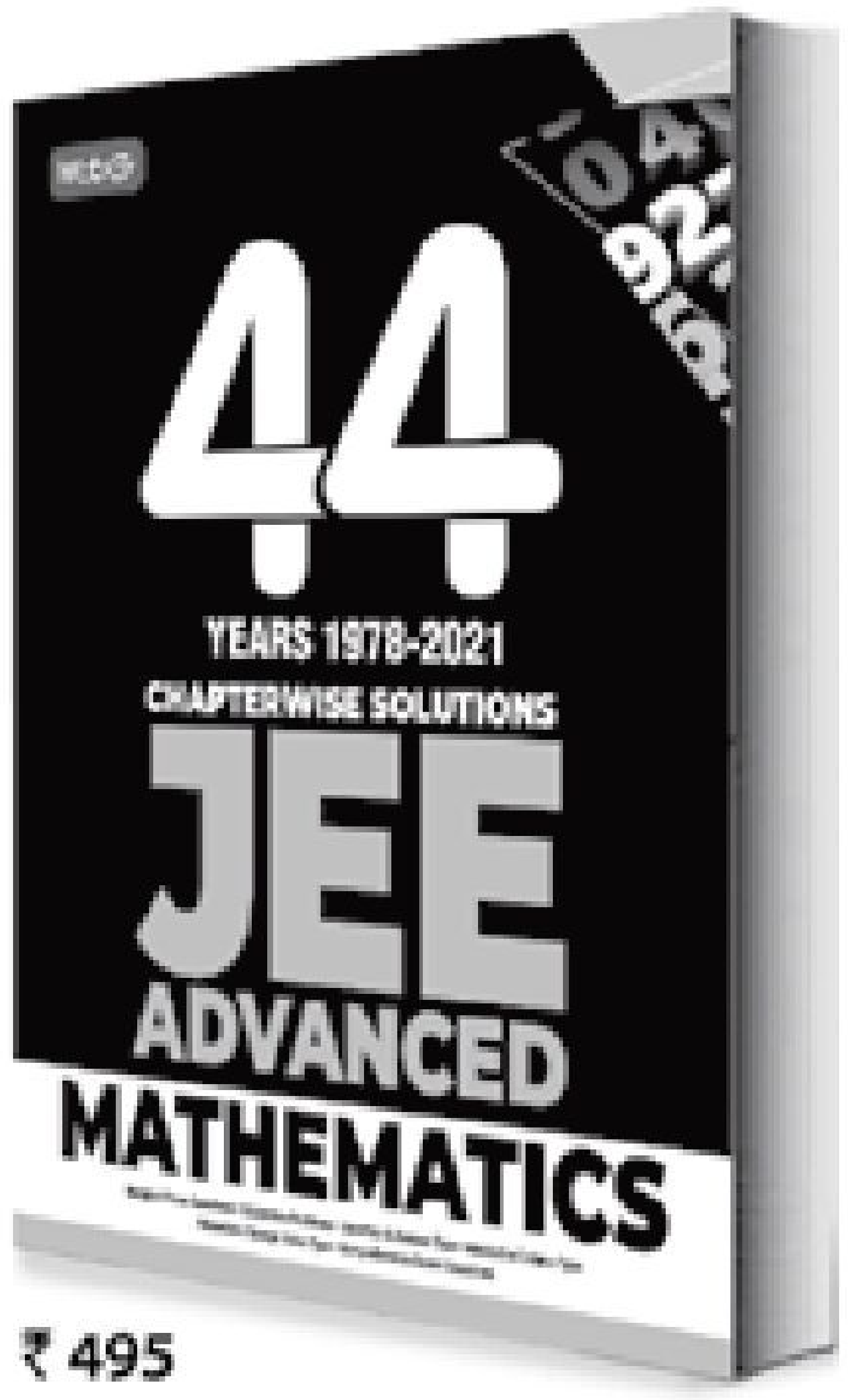
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$\Rightarrow$ a, b, c are in A.P.

$$\Rightarrow \frac{2\Delta}{p_1}, \frac{2\Delta}{p_2}, \frac{2\Delta}{p_3} \text{ are in A.P.} \Rightarrow \frac{1}{p_1}, \frac{1}{p_2}, \frac{1}{p_3} \text{ are in A.P.}$$

$\Rightarrow$ p_1, p_2, p_3 are in H.P.

15. (b) : We have, $\cos(\sin^{-1} x) = \sqrt{1-x^2}$

and $\sin(\cos^{-1} x) = \sqrt{1-x^2}$

$$\therefore \quad \tan\{\sin^{-1}(\cos(\sin^{-1}x))\}\tan\{\cos^{-1}(\sin(\cos^{-1}x))\}$$

$$= \tan\{\sin^{-1}(\sqrt{1-x^2})\}\tan\{\cos^{-1}(\sqrt{1-x^2})\}$$

$$= \tan\{\sin^{-1}(\sqrt{1-x^2})\}\tan\left\{\frac{\pi}{2} - \sin^{-1}(\sqrt{1-x^2})\right\}$$

$$= \tan\{\sin^{-1}(\sqrt{1-x^2})\}\cot\left\{\sin^{-1}(\sqrt{1-x^2})\right\} = 1$$

16. (b) : The equation of the circle described on SS' as a diameter is

$(x - ae)(x + ae) + (y - 0)(y - 0) = 0$

$\Rightarrow$ $x^2 + y^2 = a^2e^2$

The abscissa of the points of intersection of the ellipse and this circle are the roots of the equation

$$\frac{x^2}{a^2} + \frac{a^2e^2 - x^2}{b^2} = 1 \Rightarrow x^2\left(\frac{1}{a^2} - \frac{1}{b^2}\right) = 1 - \frac{a^2e^2}{b^2}$$

$$\Rightarrow x^2\left(\frac{b^2 - a^2}{a^2b^2}\right) = \frac{b^2 - a^2e^2}{b^2} \Rightarrow -\frac{x^2a^2e^2}{a^2} = a^2 - 2a^2e^2$$

$$\Rightarrow x^2 = a^2\left(\frac{2e^2 - 1}{e^2}\right) \Rightarrow x = \pm\frac{a}{e}\sqrt{2e^2 - 1}$$

This will give distinct values of x if $2e^2 - 1 > 0$

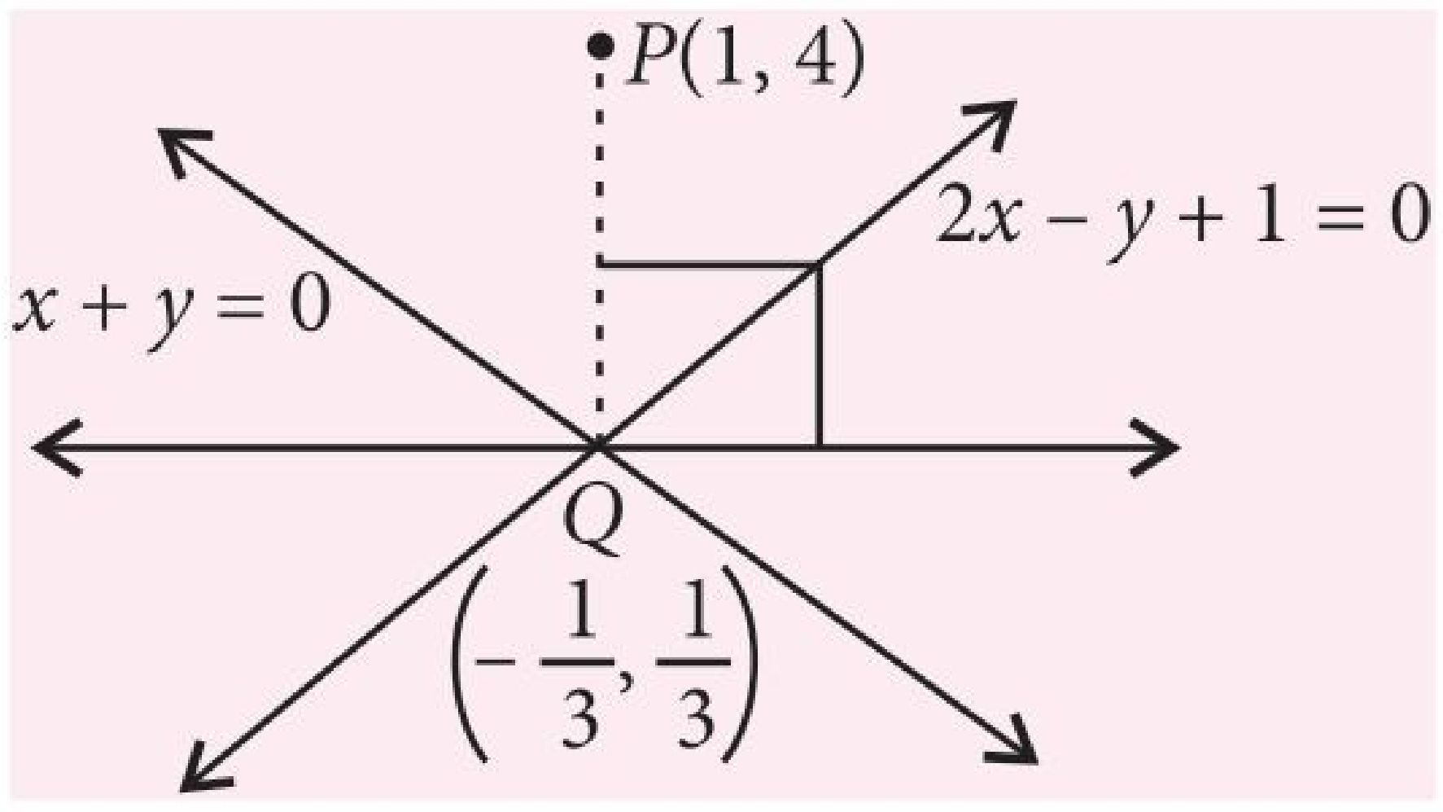
i.e. $e > \frac{1}{\sqrt{2}}$. Hence, $e \in \left(\frac{1}{\sqrt{2}}, 1\right)$

17. (c) : We have, $x + y + \lambda(2x - y + 1) = 0$ ...(i)

Clearly, (i) represents a family of lines passing through the point of intersection of the lines

$x + y = 0$ and $2x - y + 1 = 0$ *i.e.*, $\left(-\frac{1}{3}, \frac{1}{3}\right)$

The required line passes through $\left(-\frac{1}{3}, \frac{1}{3}\right)$ and is perpendicular to the line joining $(1, 4)$ and $\left(-\frac{1}{3}, \frac{1}{3}\right)$ so, its equation is



$$y - \frac{1}{3} = -\frac{4}{11}\left(x + \frac{1}{3}\right)$$

$$\Rightarrow 12x + 33y = 7$$

18. (c) : Let V be the volume of the parallelepiped formed by the given vectors.

Let $\vec{\alpha} = \hat{i} + a\hat{j} + \hat{k}, \vec{\beta} = \hat{j} + a\hat{k}, \vec{\gamma} = a\hat{i} + \hat{k}$.

$$\text{Then, } [\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 1 + a^3 - a$$

$$\therefore \ V = [\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = 1 + a^3 - a$$

$$\text{Now } \frac{dV}{da} = 3a^2 - 1 \text{ and } \frac{d^2V}{da^2} = 6a$$

For maxima or minima, we have $\frac{dV}{da} = 0 \Rightarrow a = \pm\frac{1}{\sqrt{3}}$

and $\left(\frac{d^2V}{da^2}\right)_{a=1/\sqrt{3}} = \frac{6}{\sqrt{3}} > 0$

So, V is minimum for $a = \frac{1}{\sqrt{3}}$.

19. (a) : Let $I = \int \sin(101x)\sin^{99} x\, dx$

$$\Rightarrow I = \int \sin(100x + x)\sin^{99} x\, dx$$

$$\Rightarrow I = \int \{\sin 100x \cos x + \cos 100x \sin x\}\sin^{99} x\, dx$$

$$\Rightarrow I = \int \sin 100x \cos x \sin^{99} x\, dx + \int \cos 100x \sin x \sin^{99} x\, dx$$

$$\Rightarrow I = \sin 100x \int \sin^{99} x \cos x\, dx - \int\left\{\frac{d}{dx}(\sin 100x)\int \sin^{99} x \cos x\, dx\right\}dx + \int \cos 100x \sin x \sin^{99} x\, dx$$

$$\Rightarrow I = \sin 100x \frac{\sin^{100} x}{100} - \int \cos 100x \sin^{100} x\, dx + \int \cos 100x \sin^{100} x\, dx$$

$$\Rightarrow I = \frac{1}{100}\sin(100x)(\sin x)^{100} + C$$

20. (a) : The equation of tangent at

$$P(x, y) \text{ is } Y - y = \frac{dy}{dx}(X - x)$$

It cuts the coordinate axes at

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

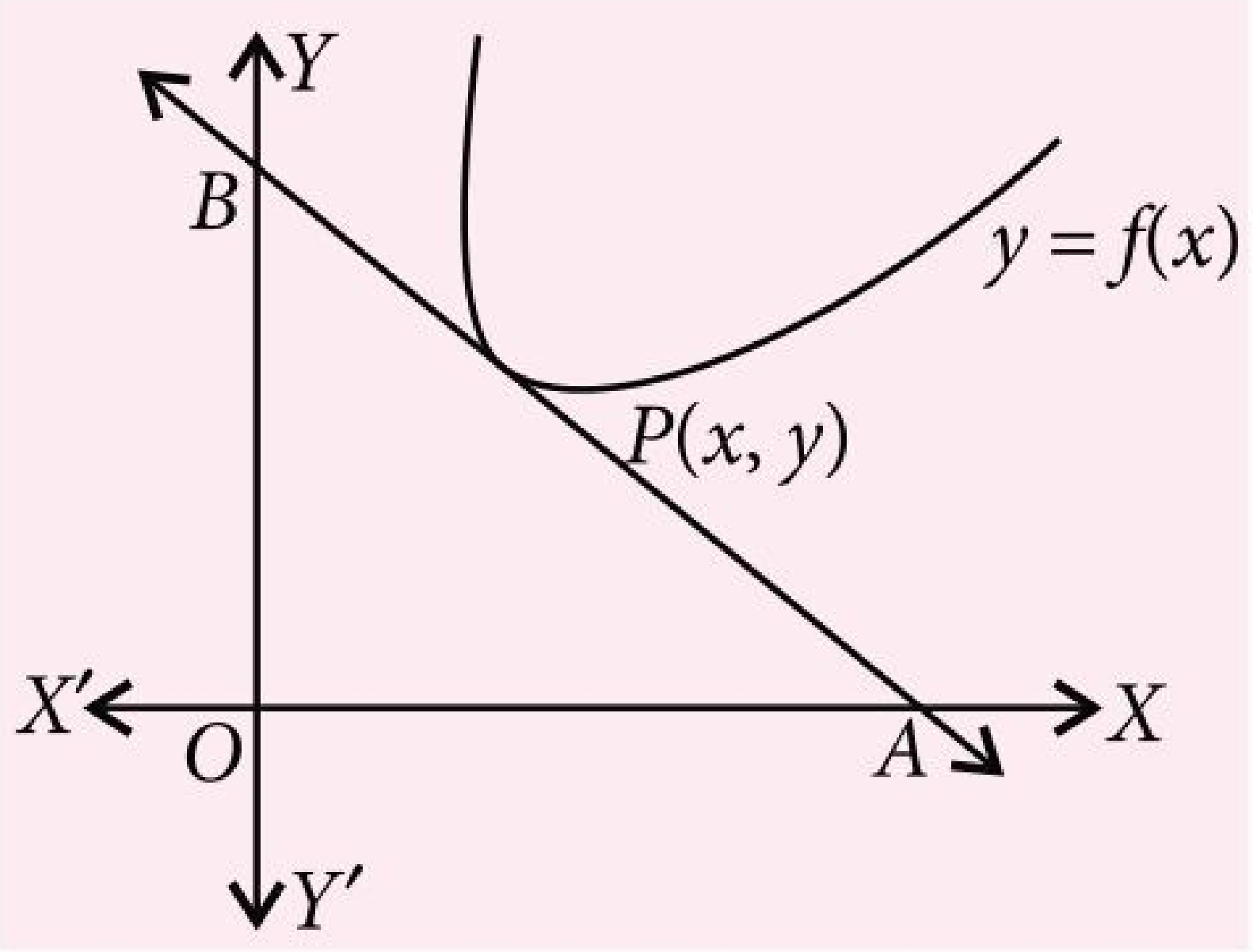
1.	(d)	**2.**	(b)	**3.**	(a)	**4.**	(d)	**5.**	(b)
6.	(d)	**7.**	(a,b)	**8.**	(d)	**9.**	(a,c)	**10.**	(b,d)
11.	(b)	**12.**	(a,b)	**13.**	(d)	**14.**	(b)	**15.**	(a)
16.	(c)	**17.**	(7)	**18.**	(6)	**19.**	(2)	**20.**	(480)

$$A\left(x - y\frac{dx}{dy}, 0\right) \text{ and } B\left(0, y - x\frac{dy}{dx}\right)$$

It is given that $AP : BP = 3 : 1$

$$\therefore \quad x = \frac{3\times 0 + x - y\dfrac{dx}{dy}}{3+1} \text{ and } y = \frac{3\left(y - x\dfrac{dy}{dx}\right)}{3+1}$$

$$\Rightarrow \quad x - y\frac{dx}{dy} = 4x \text{ and } 3y - 3x\frac{dy}{dx} = 4y$$



$$\Rightarrow \quad 3x\frac{dy}{dx} + y = 0$$

$$\Rightarrow \quad \frac{3}{y}dy + \frac{1}{x}dx = 0$$

Integrating we get,

$y^3x = C$ ...(i)

It passes through (1, 1),

$\therefore \ C = 1$

Putting $C = 1$ in (i), we get $y^3x = 1$ as the equation of the curve. Clearly, it passes through (1/8, 2).

Also, equation of normal is given by

$$Y - y = -\frac{dx}{dy}(X - x).$$

$\therefore$ Equation of normal at (1, 1) is $3x - y - 2 = 0$.

21. (c) : We have, $\vec{a}\times\vec{b} = \hat{j} - \hat{k}$

$$\Rightarrow \quad \vec{a}\times(\vec{a}\times\vec{b}) = \vec{a}\times(\hat{j} - \hat{k})$$

$$\Rightarrow (\vec{a}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{a})\vec{b} = (\hat{i} + \hat{j} + \hat{k})\times(\hat{j} - \hat{k})$$

$$\Rightarrow \quad (\hat{i}+\hat{j}+\hat{k}) - 3\vec{b} = -2\hat{i} + \hat{j} + \hat{k} \ \Rightarrow \ 3\vec{b} = 3\hat{i} \ \Rightarrow \ \vec{b} = \hat{i}$$

22. (c): We have, $x = 4t^3 + 3$ and $y = 4 + 3t^4$

$$\therefore \quad \frac{dx}{dt} = 12t^2 \text{ and } \frac{dy}{dt} = 12t^3$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{12t^3}{12t^2} = t \quad \therefore \quad \frac{dx}{dy} = \frac{1}{t} = t^{-1}$$

$$\Rightarrow \quad \frac{d^2x}{dy^2} = -t^{-2}\frac{dt}{dy} = -t^{-2}\times\frac{1}{12t^3} = -\frac{1}{12}t^{-5}$$

$$\therefore \quad \frac{d^2x}{dy^2}\bigg/\left(\frac{dx}{dy}\right)^n = -\frac{1}{12}t^{-5}\times t^n = -\frac{1}{12}t^{n-5}$$

Since $\dfrac{d^2x}{dy^2}\bigg/\left(\dfrac{dx}{dy}\right)^n$ is constant. Therefore, $n = 5$.

23. (c) : We have, $s = \sqrt{1+t}$

$$\Rightarrow \quad \frac{ds}{dt} = \frac{1}{2\sqrt{1+t}} \ \Rightarrow \ \frac{ds}{dt} = \frac{1}{2s}$$

$$\Rightarrow \quad \frac{d^2s}{dt^2} = -\frac{1}{2s^2}\cdot\frac{ds}{dt} = -\frac{1}{2s^2}\times\frac{1}{2s} = -\frac{1}{4s^3}$$

$$\Rightarrow \quad \frac{d^2s}{dt^2} = -2\times\left(\frac{1}{2s}\right)^3 = -2\left(\frac{ds}{dt}\right)^3$$

24. (d) : We know that the lines

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$$

and $\dfrac{x - x_2}{l_2} = \dfrac{y - y_2}{m_2} = \dfrac{z - z_2}{n_2}$ are coplanar, iff

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

So, the given lines will be coplanar iff

$$\begin{vmatrix} 1-2 & 4-3 & 5-4 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \ \Rightarrow \ k^2 + 3k = 0 \Rightarrow k = 0, -3.$$

25. (a) : Vectors parallel to the normals to P_1 and P_2 be $\vec{n}_1 = (2\hat{j} + 3\hat{k})\times(4\hat{j} - 3\hat{k})$ and $\vec{n}_2 = (\hat{j} - \hat{k})\times(3\hat{i} + 3\hat{j})$ respectively. Clearly $\vec{n}_1 = -18\hat{i}$ and $\vec{n}_2 = 3\hat{i} - 3\hat{j} - 3\hat{k}$

$$\therefore \quad \vec{A} = \pm(\vec{n}_1\times\vec{n}_2) = \pm 18\hat{i}\times(3\hat{i} - 3\hat{j} - 3\hat{k}) = \pm 54(\hat{j} - \hat{k})$$

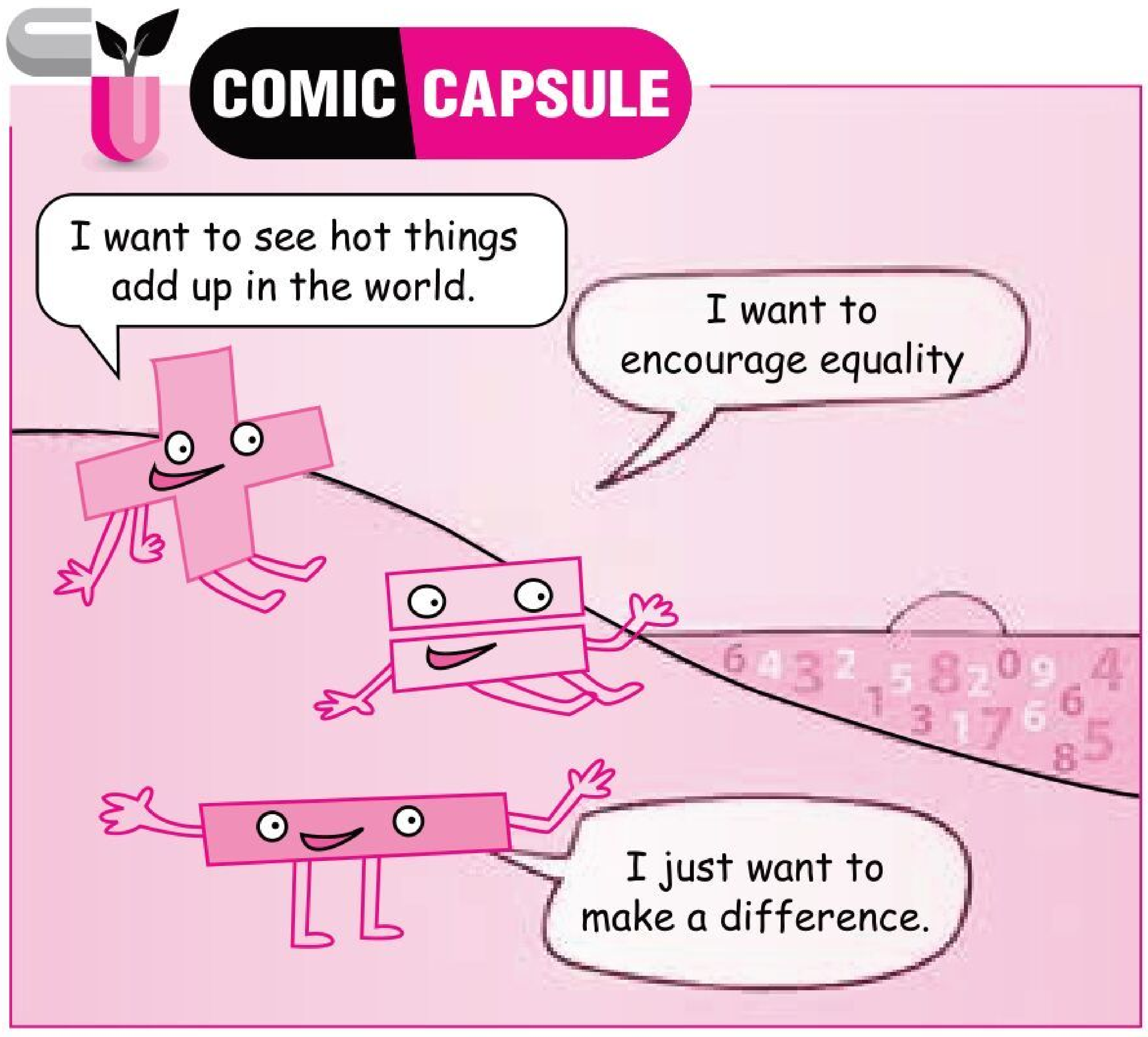
Let $\vec{B} = 2\hat{i} + \hat{j} - 2\hat{k}$

Let θ be the angle between $\vec{A}$ and $\vec{B}$.

$$\text{Then, } \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \pm\frac{54(1+2)}{3\times 54\sqrt{2}} = \pm\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \theta = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}$$

……

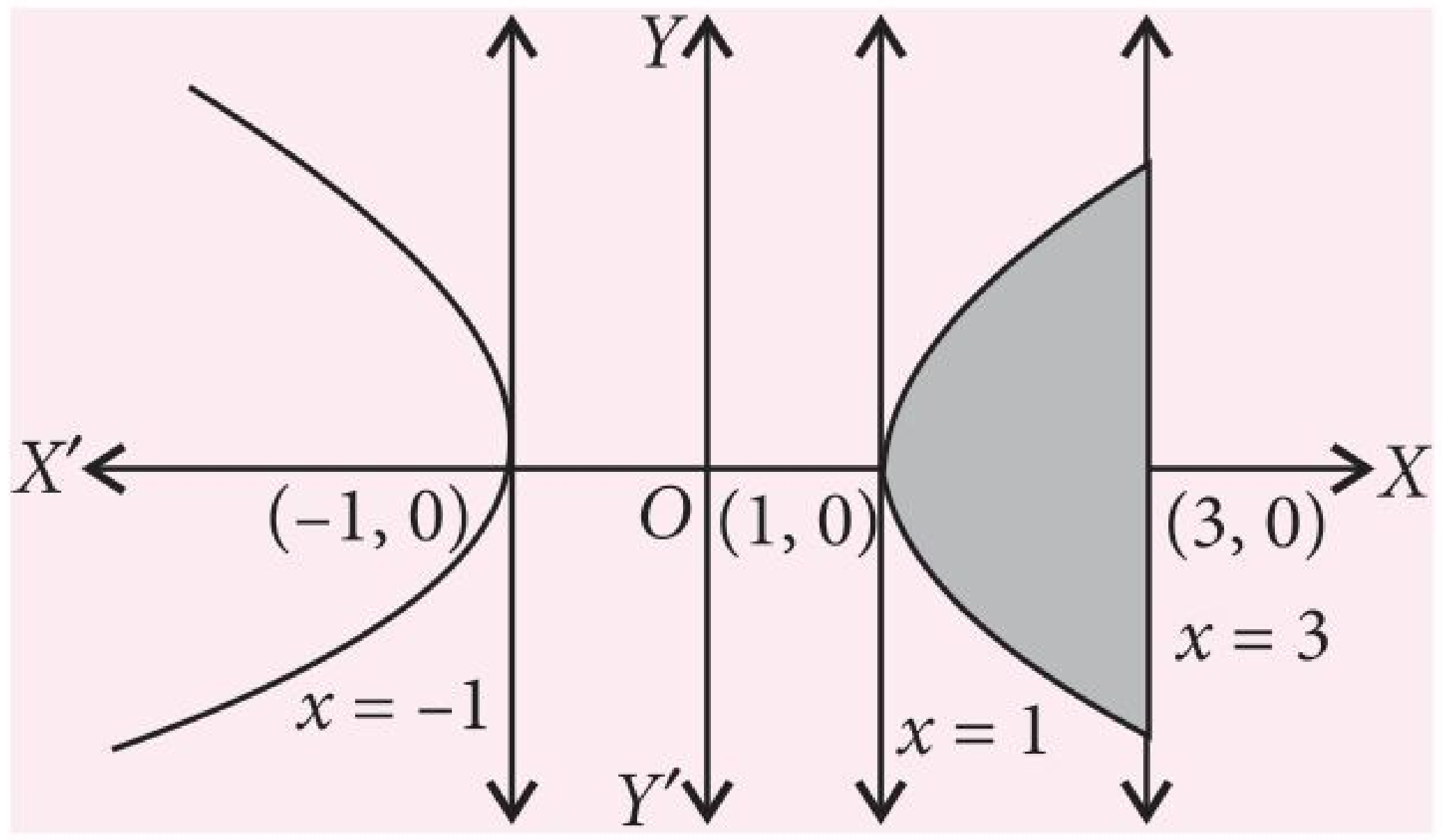


COMIC CAPSULE
I want to see hot things add up in the world.
I want to encourage equality
I just want to make a difference.

26. (c) : We have, $|x-1| \le 2 \Leftrightarrow -1 \le x \le 3$

$\therefore$ Required area A is the area of the shaded region and is given by $A = 2\int_1^3 \sqrt{x^2-1}\,dx$

$$\Rightarrow A = \left[x\sqrt{x^2-1} - \log_e\left(x+\sqrt{x^2-1}\right)\right]_1^3$$



$$= \{3\sqrt{8} - \log_e(3+\sqrt{8})\} = 6\sqrt{2} - \log_e(3+2\sqrt{2})$$

27. (c) : Mean $\bar{x} = \dfrac{a+(a+d)+\ldots+(a+2nd)}{2n+1} = a+nd$

x_i	$\lvert x_i-\bar{x}\rvert$	$\lvert x_i-\bar{x}\rvert^2$
a	nd	n^2d^2
$a+d$	$(n-1)d$	$(n-1)^2d^2$
………	………	………
………	………	………
$a+(n-2)d$	$2d$	$4d^2$
$a+(n-1)d$	d	d^2
$a+nd$	0	0
$a+(n+1)d$	d	d^2
$a+(n+2)d$	$2d$	$4d^2$
………	………	………
………	………	………
$a+2nd$	nd	n^2d^2

$$\sum|x_i-\bar{x}| = 2dn\left(\frac{n+1}{2}\right) = n(n+1)d,$$

$$\sum|x_i-\bar{x}|^2 = 2d^2[1^2+2^2+3^2\ldots\ldots+n^2]$$

$$= 2d^2\,\frac{n(n+1)(2n+1)}{6}$$

$$\text{Now, M.D.} = \frac{1}{2n+1}\sum|x_i-\bar{x}| = \frac{n(n+1)d}{2n+1}$$

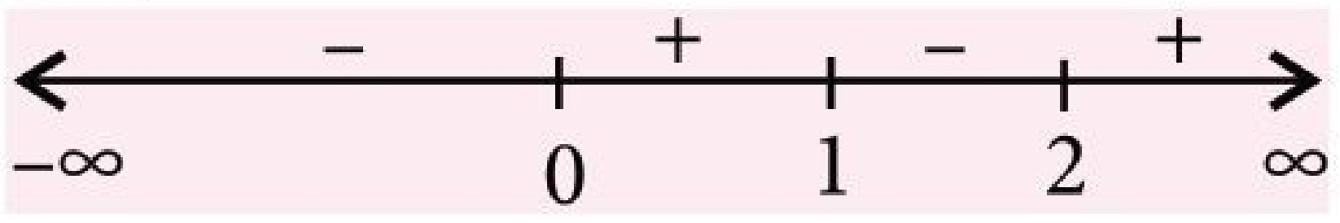
$$\text{S.D.} = \sqrt{\frac{1}{2n+1}\sum|x_i-\bar{x}|^2} = \sqrt{\frac{2d^2\cdot n(n+1)(2n+1)}{6(2n+1)}}$$

$$= \sqrt{\frac{n(n+1)d^2}{3}} = \sqrt{\frac{n(n+1)}{3}}\cdot d$$

28. (c) : We have, $f(x) = \int_1^x t(t^2-3t+2)dt$

$\Rightarrow$ $f'(x) = x^3 - 3x^2 + 2x = x(x-2)(x-1)$

The change in signs of $f'(x)$ for different values of x are as follows :



Clearly $f(x)$ is decreasing in $[1, 2]$ and increasing in $[2, 3]$.

$$\therefore \text{ Min.} f(x) = f(2) = \int_1^2 t(t^2-3t+2)dt$$

$$= \left[\frac{t^4}{4} - t^3 + t^2\right]_1^2 = -\frac{1}{4}$$

$$\text{and max.}\ f(x) = f(3) = \int_1^3 t(t^2-3t+2)dt$$

$$= \left[\frac{t^4}{4} - t^3 + t^2\right]_1^3 = 2$$

Since $f(x)$ is continuous on $[1, 3]$. Therefore, the range of $f(x) = [f(2), f(3)] = \left[-\dfrac{1}{4}, 2\right]$.

29. (c) : We have, $f'(x) = \begin{cases} 3x^2-2x+10, & x<1 \\ -2 & , x>1 \end{cases}$

Clearly, $f'(x) > 0$ for $x < 1$ and $f'(x) < 0$ for $x > 1$

Thus, $f(x)$ is increasing for all $x < 1$ and decreasing for $x > 1$.

Therefore, $f(x)$ will have greatest value at $x = 1$, if $\lim\limits_{x\to 1^+} f(x) < f(1)$ or $\lim\limits_{x\to 1^-} f(x)$

$\Rightarrow$ $\lim\limits_{x\to 1} -2x + \log_2(b^2-4) < 1-1+10-7$

$\Rightarrow$ $-2 + \log_2(b^2-4) < 3 \Rightarrow \log_2(b^2-4) < 5$

$\Rightarrow$ $b^2 - 4 < 2^5$ and $b^2 - 4 > 0$

$\Rightarrow$ $b^2 - 36 < 0$ and $b^2 - 4 > 0$

$\Rightarrow$ $-6 < b < 6$ and $b \in (-\infty, -2) \cup (2, \infty)$

$\Rightarrow$ $b \in (-6, -2) \cup (2, 6)$

30. (d) : We have, $f(x) = ax + 3\sin x + 4\cos x$

$\Rightarrow$ $f'(x) = a + 3\cos x - 4\sin x$

If $f(x)$ is invertible, then

$f'(x) > 0\ \forall\ x$ or $f'(x) < 0\ \forall\ x$

$\Rightarrow$ $a + 3\cos x - 4\sin x > 0\ \forall\ x$

or $a + 3\cos x - 4\sin x < 0\ \forall\ x$

$\Rightarrow$ $a - 5 > 0$ or $a + 5 < 0$ $\quad [\because -5 \le 3\cos x - 4\sin x \le 5]$

$\Rightarrow$ $a > 5$ or $a < -5 \Rightarrow a \in (-\infty, -5) \cup (5, \infty)$

◆◆

JEE 2022 PRACTICE PAPER ADVANCED

Exam on 3rd July

PAPER-1

SECTION 1

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options. (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*

1. The value of $\lim_{x\to 0}\left(\left[\frac{100x}{\sin x}\right]+\left[\frac{99\sin x}{x}\right]\right)$, where $[\cdot]$ denotes the greatest integer function, is

(a) 197 (b) 198
(c) 199 (d) Does not exist

2. Let $A(1, 1)$, $B(4, 2)$, $C(9, 3)$ be the vertices of a triangle ABC. A parallelogram $AFDE$ is drawn with vertices D, E, F on the line segment BC, CA, AB respectively. The maximum area of the parallelogram is

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{3}{2}$

3. Chords of the circle $x^2 + y^2 = r^2$ touch the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. The locus of the midpoints of the chords is

(a) $(x^2 + y^2)^2 = a^2x^2 - b^2y^2$
(b) $(x^2 + y^2)^2 = a^2x^2 + b^2y^2$
(c) $x^2 + y^2 = a^2x^2 - b^2y^2$
(d) $x^2 + y^2 = a^2x^2 + b^2y^2$

4. If $x = 2 + 5i$ (where $i^2 = -1$) and $2\left(\frac{1}{1!9!}+\frac{1}{3!7!}\right)+\frac{1}{5!5!}=\frac{2^a}{b!}$, then the value of $(x^3 - 5x^2 + 33x - 19)$ is equal to

(a) a (b) b
(c) $a - b$ (d) $a + b$

SECTION 2

- *This section contains THREE (03) questions stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

Question Stem for Question Nos. 5 and 6

If m, n, r are positive integers and if $r < m$, $r < n$, then
${}^mC_r + {}^mC_{r-1}\cdot{}^nC_1 + {}^mC_{r-2}\cdot{}^nC_2 + \dots + {}^nC_r$
= Coefficient of x^r in $(1 + x)^m (1 + x)^n$
= Coefficient of x^r in $(1 + x)^{m+n}$
$= {}^{m+n}C_r$

5. The value of r for which ${}^{30}C_r\cdot{}^{20}C_0 + {}^{30}C_{r-1}\cdot{}^{20}C_1 + \dots + {}^{30}C_0\cdot{}^{20}C_r$ is maximum, is

6. The value of $r(0 \le r \le 30)$ for which ${}^{20}C_r\cdot{}^{10}C_0 + {}^{20}C_{r-1}\cdot{}^{10}C_1 + \dots + {}^{20}C_0\cdot{}^{10}C_r$ is minimum, is

Question Stem for Question Nos. 7 and 8

The equations of the sides AB, BC, CA of a triangle ABC are $2x + y = 0$, $x + py = q$, $x - y = 3$ respectively. The point P is $(2, 3)$.

7. If P is the centroid, then $p + q =$

8. If P is the orthocentre, then $p + q =$

Question Stem for Question Nos. 9 and 10

Let A, B, C, D, E be the smallest positive integers having 10, 12, 15, 16, 20 positive divisors respectively. Then

9. The value of $A + B =$

10. The value of $C + D =$

SECTION 3

- *This section contains SIX (06) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*

11. The reflection of the complex number $\frac{2-i}{3+i}$, (where $i=\sqrt{-1}$) in the straight line $z(1+i)=\overline{z}(i-1)$

(a) $\frac{-1-i}{2}$ (b) $\frac{-1+i}{2}$ (c) $\frac{i(i+1)}{2}$ (d) $\frac{-1}{1+i}$

12. Equation of tangent to the curve $y = \cos(x + y)$, $-2\pi \le x \le 2\pi$, that are parallel to the line $x + 2y = 0$ is/are

(a) $x + 2y = \frac{\pi}{2}$ (b) $x + 2y = -\frac{3\pi}{2}$

(c) $x + 2y = -\frac{\pi}{2}$ (d) $x + 2y = \frac{3\pi}{2}$

13. The value of $\int_{a}^{a+\pi/2} (\sin^4 x + \cos^4 x)\,dx$ is

(a) independent of a (b) $a\left(\frac{\pi}{2}\right)^2$

(c) $\frac{3\pi}{8}$ (d) $\frac{3}{8}\pi a^2$

14. A point on the ellipse $x^2 + 3y^2 = 37$ where the normal is parallel to the line $6x = 5y$ is

(a) (5, –3) (b) (–2, 5) (c) (5, 2) (d) (–5, –2)

15. Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, $X = [a \quad 0 \quad 4]$. If $XAX^T = O$, then $a =$

(a) –4 (b) 4 (c) –12 (d) 12

16. A fair coin is tossed n times. If X is the number of times heads occur and $P(X = 4)$, $P(X = 5)$, $P(X = 6)$ are in A.P., then $n =$

(a) 13 (b) 7 (c) 11 (d) 14

SECTION 4

- *This section contains THREE (03) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*

17. If $\vec{u}, \vec{v}, \vec{w}$, are three non-coplanar unit vectors and α, β, γ are the angles between $\vec{u}$ and $\vec{v}, \vec{v}$ and $\vec{w}, \vec{w}$ and $\vec{u}$, respectively and $\vec{x}, \vec{y}, \vec{z}$ are unit vectors along the bisectors of the angles α, β, γ respectively. If $[\vec{x}\times\vec{y}\ \vec{y}\times\vec{z}\ \vec{z}\times x] = \frac{1}{\lambda}\ [\vec{u}\ \vec{v}\ \vec{w}]^2 \sec^2\frac{\alpha}{2}\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}$, then the value of $\sqrt{\lambda}$ is

18. The area enclosed by the curve $xy^2 = 4(2 - x)$ and y-axis is λ sq. units, then the value of cos (1824λ) must be

19. If $\int \tan^7 x\,dx = f(x) + \ln|\cos x| + c$, then $f(x)$ is a polynomial of degree _____ in sec x, where c is integration constant.

PAPER-2

SECTION 1

- *This section contains SIX (06) questions.*
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***

1. If $a > b > c$ and the system of equations $ax + by + cz = 0$, $bx + cy + az = 0$, $cx + ay + bz = 0$ has a non trivial solution, then both the roots of the quadratic equation $at^2 + bt + c = 0$ are

(a) real (b) of opposite sign

(c) positive (d) complex

2. Let $Q(x) = f(x) + f(1 - x)$ and $f''(x) < 0$, $0 \le x \le 1$, then

(a) Q increases in [1/2, 1]

(b) Q decreases in [1/2, 1]

(c) Q decreases in [0, 1/2]

(d) Q increases in [0,1/2]

3. Between two junction stations A and B there are 12 intermediate stations. The number of ways in which a train can be made to stop at 4 of these stations so that no two of these halting stations are consecutive, is

(a) ${}^8C_4 + 56$ (b) ${}^9C_4$

(c) ${}^{12}C_4 - 4$ (d) none of these

4. The value of $\tan\left[\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2} + \frac{1}{2}\cos^{-1}\frac{1-x^2}{1+x^2}\right]$ is

(a) $\frac{2x}{1-x^2}$ if $0 \le x \le 1$ (b) $\frac{2x}{1-x^2}$ if $x < 1$

(c) not finite if $x > 1$ (d) none of these

5. If $A = \int_0^{\pi} \frac{\sin x}{\sin x + \cos x}dx$ and $B = \int_0^{\pi} \frac{\sin x}{\sin x - \cos x}dx$ then

(a) $A = B$

(b) A and B both are rational

(c) A and B both are irrational

(d) A is period of $|\sin x|$

6. The normal $y = mx - 2am - am^3$ to the parabola $y^2 = 4ax$ subtends a right angle at the vertex if

(a) $m = 1$ (b) $m = \sqrt{2}$

(c) $m = -\sqrt{2}$ (d) $m = \frac{1}{\sqrt{2}}$

SECTION 2

- *This section contains THREE (03) question stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

Question Stem for Question Nos. 7 and 8

Let ABC be a triangle with integer sides a, b, c and $C = \frac{\pi}{2}$.

7. If $r = 3$ its greatest area, is

8. If $r = 4$ its greatest perimeter, is

Question Stem for Question Nos. 9 and 10

Let $f(x)$ be the fourth degree polynomial such that $f'(0) = -6, f(0) = 2$ and $\lim\limits_{x\to 1}\frac{f(x)}{(x-1)^2}=1$

9. The value of $f(2)$ is

10. The value of $f'(2)$ is

Question Stem for Question Nos. 11 and 12

If a particular experiment be given n(a finite) independent trials.

If the probability of success in one trial (say)p

$\therefore$ We get probability of failure, $q = (1 - p)$

The probability of r success in n trials = ${}^nC_r p^r q^{n-r}$

11. The probability of a man hitting a targent in one fire is $\frac{1}{4}$. Atleast n times he must fire at the target that the chances of hitting the target at least once will exceed $\frac{2}{3}$, then n is _______.

12. When two dice are thrown n number of times, the probability of getting atleast one double six is greater than 99% then the least value of n is _______.
(Given log 35 = 1.5441, log 36 = 1.5563)

SECTION 3

- *This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO(02) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*
- *For each question, choose the option corresponding to the correct answer.*

Paragraph for Q. 13 and Q. 14

Let V_r denote the sum of first r terms of an A.P., whose first term is r and the common difference is $(2r - 1)$. Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1, 2, \ldots$

13. The sum $V_1 + V_2 + \ldots. + V_n$ is

(a) $\frac{1}{12}n(n+1)(3n^2-n+1)$

(b) $\frac{1}{12}n(n+1)(3n^2+n+2)$

(c) $\frac{1}{2}n(2n^2-n+1)$ (d) $\frac{1}{3}(2n^3-2n+3)$

14. T_r is always

(a) an odd number (b) an even number
(c) a prime number (d) a composite number

Paragraph for Q. 15 and Q. 16

Let $f(x)$ be a continuous function defined on the closed interval $[a, b]$, then

$$\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{n}f\left(\frac{r}{n}\right)=\int_0^1 f(x)\,dx$$

15. The value of $\lim\limits_{n\to\infty}\frac{1}{n}\left\{\frac{1}{n+1}+\frac{2}{n+2}+\ldots+\frac{3n}{4n}\right\}$ is

(a) 5 – 2ln 2 (b) 4 – 2ln 2
(c) 3 – 2ln 2 (d) 2 – 2ln 2

16. The value of $\lim\limits_{n\to\infty}\left\{\prod_{r=1}^{n}\left(1+\frac{r}{n}\right)\right\}^{1/n}$ is

(a) $\frac{3}{e}$ (b) $\frac{4}{e}$ (c) $\frac{1}{e}$ (d) $\frac{2}{e}$

SOLUTIONS

PAPER-1

1. (b) : In the neighbourhood of 0, $x > \sin x$

$$\Rightarrow \frac{\sin x}{x}<1 \quad \therefore \quad \frac{99\sin x}{x}<99 \Rightarrow \left[\frac{99\sin x}{x}\right]=98$$

$$\text{and}\quad \frac{x}{\sin x}>1 \quad \therefore \quad \frac{100x}{\sin x}>100 \Rightarrow \left[\frac{100x}{\sin x}\right]=100$$

$$\therefore \lim_{x\to 0}\left(\left[\frac{100x}{\sin x}\right]+\left[\frac{99\sin x}{x}\right]\right)=100+98=198$$

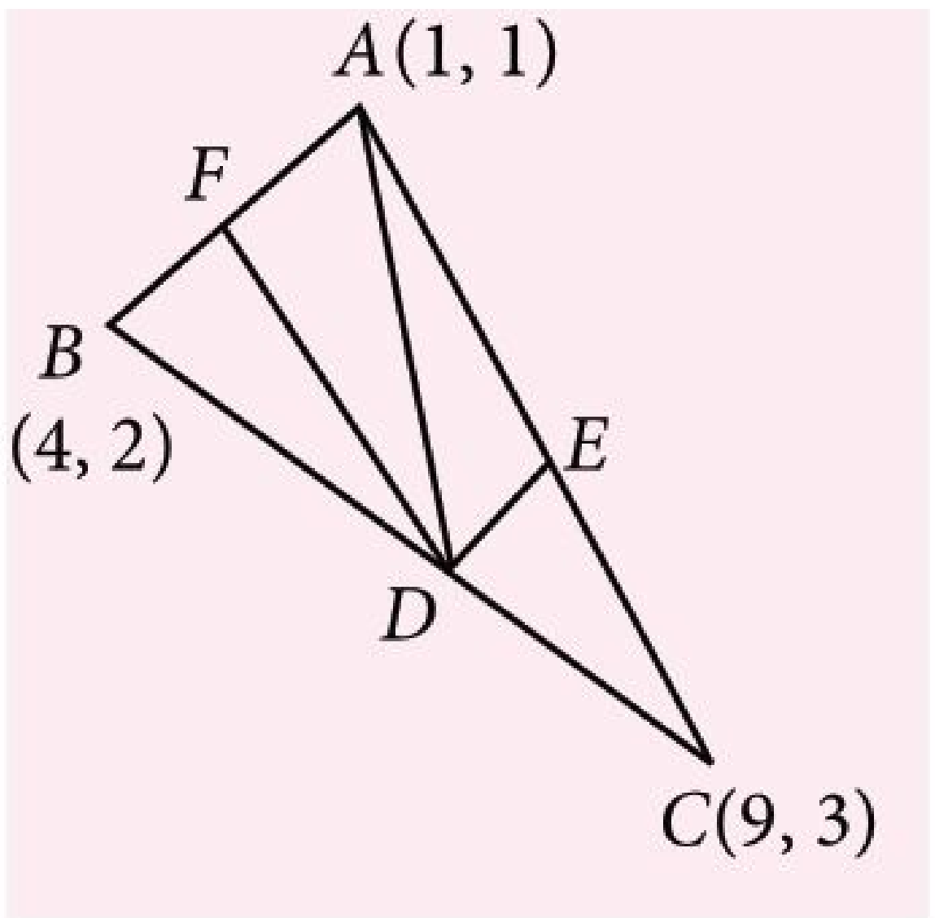
2. (b) : Let D divides BC in the ratio $\lambda : 1$. Then

$$D=\left(\frac{4+9\lambda}{1+\lambda},\frac{2+3\lambda}{1+\lambda}\right), E=\left(\frac{1+9\lambda}{1+\lambda},\frac{1+3\lambda}{1+\lambda}\right)$$

Area of $AFDE = 2\,\Delta AED$.

$$(1+\lambda)^2\Delta=\begin{vmatrix}1 & 1 & 1\\ 1+9\lambda & 1+3\lambda & 1+\lambda\\ 4+9\lambda & 2+3\lambda & 1+\lambda\end{vmatrix}$$

Applying $R_3 \to R_3 - (\lambda + 1) R_1$ and $R_2 \to R_2 - (\lambda + 1) R_1$, we get



$$(1+\lambda)^2\Delta=\begin{vmatrix}1 & 1 & 1\\ 8\lambda & 2\lambda & 0\\ 3+8\lambda & 1+2\lambda & 0\end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, we get

$$(1+\lambda)^2\Delta=\begin{vmatrix}1 & 1 & 1\\ 8\lambda & 2\lambda & 0\\ 3 & 1 & 0\end{vmatrix}=2\lambda$$

$$\therefore\quad \Delta=\frac{2\lambda}{(\lambda+1)^2}=\frac{2}{\left(\sqrt{\lambda}+\frac{1}{\sqrt{\lambda}}\right)^2}\le\frac{2}{4}=\frac{1}{2}.$$

3. (a) : Let $P(x_1, y_1)$ be the midpoint of a chord of $x^2 + y^2 = r^2$.

Its equation is $S_1 = S_{11} \Rightarrow x_1x + y_1y = x_1^2 + y_1^2$

or $y=\left(-\frac{x_1}{y_1}\right)x+\frac{x_1^2+y_1^2}{y_1}$

The line $y = mx + c$ is a tangent to the hyperbola only if $c^2 = a^2m^2 - b^2$

$$\therefore\quad \frac{(x_1^2+y_1^2)^2}{y_1^2}=\frac{a^2x_1^2}{y_1^2}-b^2$$

$\therefore$ The locus of P is $(x^2 + y^2)^2 = a^2x^2 - b^2y^2$.

4. (b) : $2\left(\frac{1}{1!9!}+\frac{1}{3!7!}\right)+\frac{1}{5!5!}=\frac{2^a}{b!}$

$$\Rightarrow \frac{2}{1!9!}+\frac{2}{3!7!}+\frac{1}{5!5!}=\frac{2^a}{b!}$$

$$\Rightarrow \frac{1}{10!}\left(\frac{2!10!}{1!9!}+\frac{2!10!}{3!\,7!}+\frac{10!}{5!5!}\right)=\frac{2^a}{b!}$$

$$\Rightarrow \frac{1}{10!}({}^{10}C_1+{}^{10}C_1+{}^{10}C_3+{}^{10}C_3+{}^{10}C_5)=\frac{2^a}{b!}$$

$$\Rightarrow \frac{1}{10!}({}^{10}C_1+{}^{10}C_9+{}^{10}C_3+{}^{10}C_7+{}^{10}C_5)=\frac{2^a}{b!}$$

$$\Rightarrow \frac{1}{10!}({}^{10}C_1+{}^{10}C_3+{}^{10}C_5+{}^{10}C_7+{}^{10}C_9)=\frac{2^a}{b!}$$

$$\Rightarrow \frac{2^{10-1}}{10!}=\frac{2^a}{b!}$$

$\therefore\quad a = 9,\ b = 10$

Also, $x = 2 + 5i$

$\therefore\quad (x - 2)^2 = (5i)^2 \Rightarrow x^2 - 4x + 29 = 0$

$\therefore\quad x^3 - 5x^2 + 33x - 19 = (x - 1)(x^2 - 4x + 29) + 10$

$= 0 + 10 \qquad (\because x^2 - 4x + 29 = 0)$

$= 10 = b.$

5. (25) : $\because {}^{30}C_r \cdot {}^{20}C_0 + {}^{30}C_{r-1}\cdot {}^{20}C_1 + \ldots + {}^{30}C_0\cdot{}^{20}C_r$

= coefficient of x^r in $(1 + x)^{30}(1 + x)^{20}$

= coefficient of x^r in $(1 + x)^{50} = {}^{50}C_r$

$\because\quad {}^{50}C_r$ is maximum

$\therefore\quad r=\frac{50}{2}=25$

6. (0) : $\because {}^{20}C_r \cdot {}^{10}C_0 + {}^{20}C_{r-1} \cdot {}^{10}C_1 + \ldots + {}^{20}C_0 \cdot {}^{10}C_r$

= coefficient of x^r in $(1 + x)^{20}(1 + x)^{10}$

= coefficient of x^r in $(1 + x)^{30}$

$= {}^{30}C_r$

$\because\quad {}^{30}C_r$ is minimum

$\therefore\quad r = 0$

7. (74) : $A \equiv (1, -2)$, $B \equiv (\alpha, -2\alpha)$, $C \equiv (\beta, \beta - 3)$

$1 + \alpha + \beta = 6,\ -2 - 2\alpha + \beta - 3 = 9$

$\Rightarrow\quad \alpha = -3,\ \beta = 8,\ \therefore B \equiv (-3, 6),\ C \equiv (8, 5)$

Equation of BC is $x + 11y = 63$

$\therefore\quad p + q = 63 + 11 = 74$

8. (50) : Slope of $BP = -1$

$$\Rightarrow\ \frac{3+2\alpha}{2-\alpha}=-1\Rightarrow\alpha=-5$$

Slope of $CP = 1/2 \Rightarrow \frac{\beta-6}{\beta-2}=\frac{1}{2}\Rightarrow\beta=10$

$\therefore\quad B(-5, 10),\ C(10, 7)$

Equation of BC : $x + 5y = 45$

$\therefore\quad p + q = 50.$

Solution (9-10) :

$10 = 2 \cdot 5 \Rightarrow A = 2^4 \cdot 3 = 48$

$12 = 2 \cdot 2 \cdot 3 \Rightarrow B = 2^2 \cdot 3 \cdot 5 = 60$

$15 = 3 \cdot 5 \Rightarrow C = 2^4 \cdot 3^2 = 144$

$16 = 2 \cdot 2 \cdot 2 \cdot 2 \Rightarrow D = 2 \cdot 3 \cdot 5 \cdot 7 = 210$

$20 = 2 \cdot 2 \cdot 5 \Rightarrow E = 2^4 \cdot 3 \cdot 5 = 240$

9. (108) : $A + B = 48 + 60 = 108$

10. (354) : $C + D = 144 + 210 = 354$

11. (b, c, d) : $\frac{2-i}{3+i}=\frac{(2-i)(3-i)}{10}=\frac{5-5i}{10}=\frac{1}{2}-\frac{i}{2}$

i.e., $\left(\frac{1}{2},-\frac{1}{2}\right)$ and $z(1+i)=\overline{z}(i-1)$

$$\Rightarrow (z+\overline{z})+i(z-\overline{z})=0\Rightarrow\left(\frac{z+\overline{z}}{2}\right)+i\left(\frac{z-\overline{z}}{2}\right)=0$$

$\Rightarrow\quad x + i(iy) = 0\ (\because z = x + iy)$

$\Rightarrow\quad x - y = 0.$

$\therefore$ Reflection of $\left(\frac{1}{2},-\frac{1}{2}\right)$ w.r.t. $y = x$ is $\left(-\frac{1}{2},\frac{1}{2}\right)$

i.e., $-\frac{1}{2}+\frac{i}{2}=\frac{-1+i}{2}$ (Option (b))

$=\frac{i^2+i}{2}=\frac{i(i+1)}{2}$ (Option (c))

$=\frac{i(1+i)^2}{2(1+i)}=\frac{i(1+i^2+2i)}{2(1+i)}=\frac{i^2}{1+i}=\frac{-1}{1+i}$ (Option (d))

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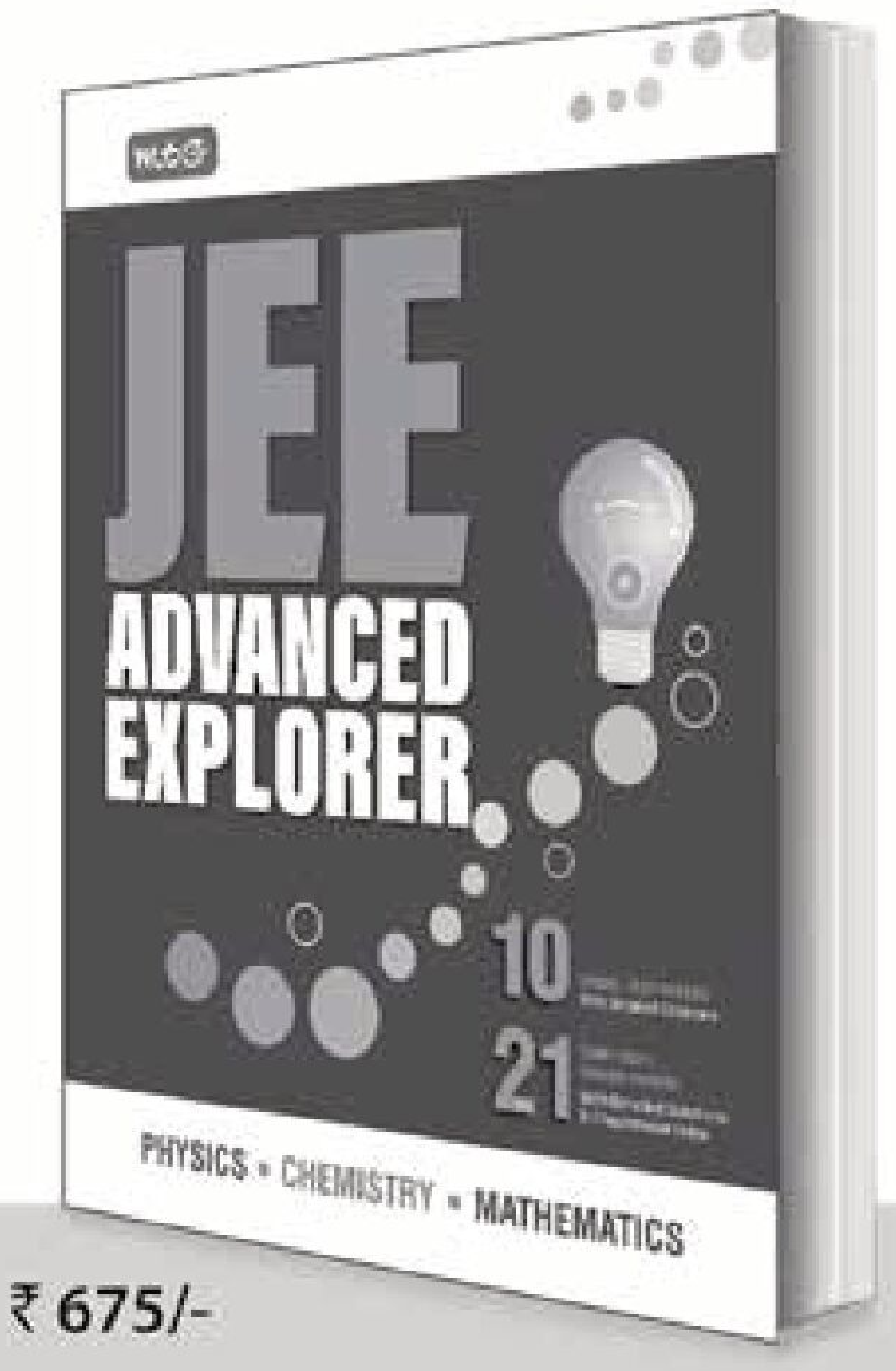
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12. (a, b) : Slope of given line = $-\frac{1}{2}$

$\Rightarrow$ Slope of tangent = $\left(\frac{dy}{dx}\right) = -\frac{1}{2}$ ... (i)

The equation of given curve is $y = \cos(x + y)$

Differentiating the curve w.r.t. x, we get

$$\frac{dy}{dx} = -\sin(x+y)\left\{1+\frac{dy}{dx}\right\}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sin(x+y)}{1+\sin(x+y)}$$

= slope of tangent ...(ii)

From (i) and (ii), $\frac{-\sin(x+y)}{1+\sin(x+y)} = -\frac{1}{2}$

$\Rightarrow$ $\sin(x + y) = 1$ ...(iii)

$\Rightarrow$ $\cos(x + y) = 0$

From the given curve $y = \cos(x + y) \Rightarrow y = 0$ ...(iv)

and $\sin(x) = 1$ [Using (iii) and (iv)]

$$\Rightarrow x = \frac{\pi}{2}, -\frac{3\pi}{2}$$

$\therefore$ The points are $P(\pi/2, 0)$, $Q(-3\pi/2, 0)$

Tangent at P is $x+2y = \frac{\pi}{2}$ and tangent at Q is $x+2y = -\frac{3\pi}{2}$

13. (a, c): Let $f(x) = \sin^4 x + \cos^4 x$, if is easy to show that $f(x)$ is periodic with period $\pi/2$

$$\therefore \int_a^{a+\pi/2} (\sin^4 x + \cos^4 x)\,dx$$

$$= \int_0^{\pi/2} (\sin^4 x + \cos^4 x)\,dx \quad \text{(By property)}$$

$$= \frac{3\cdot 1}{4\cdot 2}\cdot\frac{\pi}{2} + \frac{3\cdot 1}{4\cdot 2}\cdot\frac{\pi}{2} \quad \text{(By Walli's formula)}$$

$$= \frac{3\pi}{8}$$

14. (c, d) : Given, $\frac{x^2}{37} + \frac{y^2}{37/3} = 1$

The normal at $\left(\sqrt{37}\cos\theta, \sqrt{\frac{37}{3}}\sin\theta\right)$ is

$$\frac{\sqrt{37}x}{\cos\theta} - \sqrt{\frac{37}{3}}\frac{y}{\sin\theta} = 37 - \frac{37}{3} = \frac{74}{3}$$

Its slope is $\sqrt{3}\tan\theta = \frac{6}{5}$,

as it is parallel to $6x = 5y$.

$$\therefore \tan\theta = \frac{2\sqrt{3}}{5}, \sin\theta = \pm\frac{2\sqrt{3}}{\sqrt{37}}, \cos\theta = \pm\frac{5}{\sqrt{37}}$$

$\therefore$ Points are $\left(\sqrt{37}\cos\theta, \sqrt{\frac{37}{3}}\sin\theta\right)$ *i.e.*,

(5, 2) and (–5, –2).

15. (a, c) : $X^T = \begin{bmatrix} a \\ 0 \\ 4 \end{bmatrix}$; $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

$$\text{Now, } AX^T = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} a \\ 0 \\ 4 \end{bmatrix} \Rightarrow AX^T = \begin{bmatrix} a+8 \\ 4 \\ 2a+12 \end{bmatrix}$$

$$\text{Now, } XAX^T = \begin{bmatrix} a & 0 & 4 \end{bmatrix}\begin{bmatrix} a+8 \\ 4 \\ 2a+12 \end{bmatrix} = a^2 + 16a + 48$$

Now, $XAX^T = O$

$\Rightarrow$ $a^2 + 16a + 48 = 0$ $\Rightarrow$ $(a + 12)(a + 4) = 0$

$\Rightarrow$ $a = -12, a = -4$

16. (b, d): $\binom{n}{4}\frac{1}{2^n}, \binom{n}{5}\frac{1}{2^n}, \binom{n}{6}\frac{1}{2^n}$ are in A.P.

$\Rightarrow$ $\binom{n}{4}, \binom{n}{5}, \binom{n}{6}$ are in A.P.

$$\Rightarrow 2\binom{n}{5} = \binom{n}{4} + \binom{n}{6}$$

Dividing by $\binom{n}{5}$, $2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5} = \frac{5}{n-4} + \frac{n-5}{6}$

$$= \frac{n^2 - 9n + 50}{6(n-4)}$$

$\Rightarrow n^2 - 21n + 98 = 0 \Rightarrow n = 7, 14.$

17. (4) : We know that $[\vec{x}\times\vec{y}\;\; \vec{y}\times\vec{z}\;\; \vec{z}\times\vec{x}] = [\vec{x}\;\vec{y}\;\vec{z}]^2$

Also a vector along the bisector of given two unit vectors $\vec{u}$, $\vec{v}$ is $\vec{u}+\vec{v}$.

A unit vector along the bisector is $\frac{\vec{u}+\vec{v}}{|\vec{u}+\vec{v}|}$.

$$|\vec{u}+\vec{v}|^2 = 1 + 1 + 2\vec{u}\cdot\vec{v} = 2 + 2\cos\alpha = 4\cos^2\frac{\alpha}{2}$$

$$\Rightarrow \vec{x} = \frac{\vec{u}+\vec{v}}{2\cos\frac{\alpha}{2}}$$

Similarly, $\vec{y}=\dfrac{\vec{v}+\vec{w}}{2\cos(\beta/2)}$ and $\vec{z}=\dfrac{\vec{u}+\vec{w}}{2\cos(\gamma/2)}$

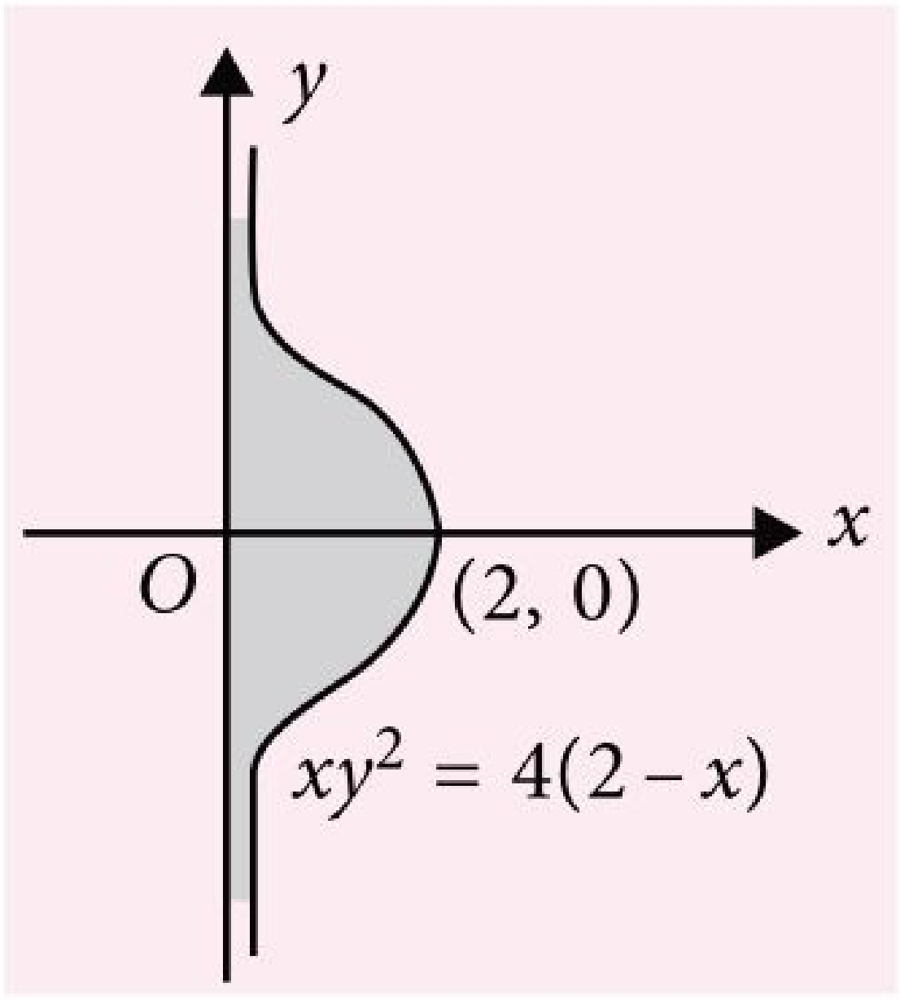
$$\Rightarrow\ [\vec{x}\,\vec{y}\,\vec{z}]=\frac{1}{8}[\vec{u}+\vec{v}\ \ \vec{v}+\vec{w}\ \ \vec{u}+\vec{w}]\times\sec\frac{\alpha}{2}\sec\frac{\beta}{2}\sec\frac{\gamma}{2}$$

$$=\frac{1}{8}2[\vec{u}\,\vec{v}\,\vec{w}]\sec\frac{\alpha}{2}\sec\frac{\beta}{2}\sec\frac{\gamma}{2}=\frac{1}{4}[\vec{u}\,\vec{v}\,\vec{w}]\sec\frac{\alpha}{2}\sec\frac{\beta}{2}\sec\frac{\gamma}{2}$$

Now, $[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times\vec{x}]=[\vec{x}\,\vec{y}\,\vec{z}]^2$

$$=\frac{1}{16}[\vec{u}\,\vec{v}\,\vec{w}]^2\sec^2\frac{\alpha}{2}\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}$$

18. (1) : The curve $xy^2 = 4(2 - x)$ is symmetrical about x-axis and at $x \to 0$, $y \to \infty$, *i.e.*, y-axis is asymptote and cuts x-axis at $x = 2$.



$$\because\ \ y^2=\frac{4(2-x)}{x}\ge 0$$

$$\Rightarrow\ \ \frac{2-x}{x}\ge 0\Rightarrow\frac{(x-2)}{x}\le 0$$

$\therefore\ \ x \in (0, 2]$

Then, $\lambda=2\displaystyle\int_0^2 2\sqrt{\left(\frac{2-x}{x}\right)}dx$

Put $x = 2\sin^2\theta$, so that $dx = 4\sin\theta\cos\theta\, d\theta$

$$\therefore\ \ \lambda=4\int_0^{\pi/2}\sqrt{\left(\frac{2-2\sin^2\theta}{2\sin^2\theta}\right)}\times 4\sin\theta\cos\theta d\theta$$

$$=16\int_0^{\pi/2}\cos^2\theta d\theta=16.\frac{1}{2}.\frac{\pi}{2}=4\pi$$

Then, $\cos(1824\lambda) = \cos(1824 \times 4\pi) = 1$

19. (6) : Let $I=\displaystyle\int\tan^7 x\,dx=\int\frac{\sin^7 x}{\cos^7 x}dx$

$$=\int\frac{(1-\cos^2 x)^3}{\cos^7 x}\cdot\sin x\,dx$$

Put $\cos x = t$

$\therefore\ \ -\sin x\, dx = dt \Rightarrow \sin x dx = -dt$

Then, $I=-\displaystyle\int\frac{(1-t^2)^3}{t^7}dt=-\int\left(\frac{1-t^6+3t^2-3t^4}{t^7}\right)dt$

$$=-\int\left(t^{-7}-\frac{1}{t}+3t^{-5}-3t^{-3}\right)dt$$

$$=-\left\{\frac{t^{-6}}{-6}-\ln|t|+\frac{3t^{-4}}{-4}-3\frac{t^{-2}}{-2}\right\}+c$$

$$=\frac{1}{6}\sec^6 x+\frac{3}{4}\sec^4 x-\frac{3}{2}\sec^2 x+\ln|\cos x|+c$$

On comparing, we get

$$f(x)=\frac{1}{6}\sec^6 x+\frac{3}{4}\sec^4 x-\frac{3}{2}\sec^2 x$$

Hence, $f(x)$ is a polynomial of degree 6 in sec x.

PAPER-2

1. (a, b): $\because\ a > b > c$ and given equations are

$ax + by + cz = 0$ $\qquad$ $bx + cy + az = 0$

$cx + ay + bz = 0$

For non trivial solution

$$\Delta=\begin{vmatrix} a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}=0$$

$3abc - (a^3 + b^3 + c^3) = 0$

$\therefore\ \ a + b + c = 0$

If α, β be the roots of $at^2 + bt + c = 0$

$\therefore\ \ \alpha+\beta=-\dfrac{b}{a}$ and $\alpha\beta=\dfrac{c}{a}$

$b^2 - 4ac = (-a - c)^2 - 4ac \ = (a - c)^2 > 0$

For opposite sign $|\alpha - \beta| > 0$

$\therefore\ \ (\alpha + \beta)^2 - 4\alpha\beta > 0$

$\dfrac{b^2}{a^2}-\dfrac{4c}{a}>0$ $\qquad$ $b^2 - 4ac > 0$

$(-a - c)^2 - 4ac > 0$ $\qquad$ $(a - c)^2 > 0$ true

Hence, roots are real and have opposite sign.

2. (b, d) : $\therefore\ f''(x) < 0$ in $0 \le x \le 1$

$\Rightarrow\ \ f'(x)$ is decreasing function.

Now, $Q'(x) = f'(x) - f'(1 - x)$

Case I: If $x \ge (1 - x)$

$\therefore\ \ f'(x) \le f'(1 - x)$

$\Rightarrow\ \ f'(x) - f'(1 - x) \le 0$ $\qquad$ $\Rightarrow Q'(x) \le 0$

$\therefore\ \ Q(x)$ is decreasing, when $x \ge 1 - x$ and $0 \le x \le 1$

$\therefore\ \ x\in\left[\dfrac{1}{2},1\right].$

EXAM ALERT 2022

Exam	Date
JEE Main	Session 1 : 21st, 24th, 25th, 29th April and 1st, 4th May Session 2 : 24th to 29th May
JEE Advanced	3rd July
SRMJEEE	Phase 2 : 23rd and 24th April Phase 3 : 25th and 26th June
WB JEE	23rd April

Case II: If $x \geq (1 - x)$

$\therefore \quad f'(x) \geq f'(1 - x)$

$\Rightarrow \quad f'(x) - f'(1 - x) \geq 0 \qquad \Rightarrow Q'(x) \geq 0$

$\therefore \quad Q(x)$ is increasing, when $x \leq 1 - x$ and $0 \leq x \leq 1$

$\therefore \quad x \in \left[0, \frac{1}{2}\right].$

3. (a, b) : Let x_1 be the number of stations before the first halting station, x_2 between first and second, x_3 between second and third, x_4 between third and fourth and x_5 on the right of the 4th station. Then,
$x_1 \geq 0, x_5 \geq 0, x_2, x_3, x_4 \geq 1$ such that
$x_1 + x_2 + x_3 + x_4 + x_5 = 8$
The total number of ways is the number of solutions of the above equation.
Let $X_2 = x_2 - 1, X_3 = x_3 - 1, X_4 = x_4 - 1$. Then,
$x_1 + X_2 + X_3 + X_4 + x_5 = 5$, where $X_2, X_3, X_4 \geq 0$.
The number of solutions of this equation is
${}^{5+5-1}C_{5-1} = {}^{9}C_4$.

4. (a, c) : Let $x = \tan\theta$, then the expression becomes

$$\tan\left\{\frac{1}{2}\sin^{-1}\sin 2\theta + \frac{1}{2}\cos^{-1}\cos 2\theta\right\}$$

If $0 \leq 2\theta \leq \frac{\pi}{2}$, the expression becomes

$$\tan\left(\frac{1}{2}\cdot 2\theta + \frac{1}{2}\cdot 2\theta\right) = \tan 2\theta = \frac{2x}{1 - x^2}$$

But $0 \leq 2\theta \leq \frac{\pi}{2} \Rightarrow 0 \leq \theta \leq \frac{\pi}{4} \Rightarrow 0 \leq x \leq 1$

If $\frac{\pi}{2} < 2\theta \leq \pi$, *i.e.*, $\tan\theta > 1$, *i.e.*, $x > 1$ then the expression

$$= \tan\left\{\frac{1}{2}\sin^{-1}\sin(\pi - 2\theta) + \frac{1}{2}\cos^{-1}\cos 2\theta\right\}$$

$$= \tan\left\{\frac{\pi - 2\theta}{2} + \frac{1}{2}\cdot 2\theta\right\} = \tan\frac{\pi}{2} = \infty$$

5. (a, c) : $A = \int_0^{\pi} \frac{\sin x}{\sin x + \cos x}dx$

$$= \int_0^{\pi} \frac{\sin(\pi - x)}{\sin(\pi - x) + \cos(\pi - x)}dx = \int_0^{\pi} \frac{\sin x}{\sin x - \cos x}dx = B$$

Now

$$A + B = \int_0^{\pi} \frac{\sin x(\sin x - \cos x) + \sin x(\sin x + \cos x)}{(\sin x - \cos x)(\sin x + \cos x)}dx$$

$$= \int_0^{\pi} \frac{2\sin^2 x}{(\sin^2 x - \cos^2 x)}dx = \int_0^{\pi} \frac{1 - \cos 2x}{-\cos 2x}dx$$

$$= \int_0^{\pi} (1 - \sec 2x)\,dx = \left[x - \frac{1}{2}\log|\sec 2x + \tan 2x|\right]_0^{\pi}$$

$$= \pi - \frac{1}{2}\log|\sec 2\pi + \tan 2\pi| + \frac{1}{2}\log|\sec 0 + \tan 0| = \pi$$

$\therefore \quad A = B = \frac{\pi}{2}$

6. (b, c): Making $y^2 = 4ax$ homogeneous with the help of $y = mx - 2am - am^3$

or $\quad \frac{mx - y}{2am + am^3} = 1$

Then $\quad y^2 = 4ax\left(\frac{mx - y}{2am + am^3}\right)$

$\Rightarrow \quad y^2 = \frac{4x(mx - y)}{2m + m^3}$

$\Rightarrow \quad (2m + m^3)y^2 - 4mx^2 + 4xy = 0 \qquad$ ...(i)

$\because \quad$ Angle between the lines represented by eqn (i) is $\pi/2$

$\therefore \quad$ Coefficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow \quad 2m + m^3 - 4m = 0$

$\Rightarrow \quad m^3 - 2m = 0$

$\quad m^2 = 2, m \neq 0$

$\therefore \quad m = \pm\sqrt{2}$

7. (84) : $(a - 2r)(b - 2r) = 2r^2 \Rightarrow (a - 6)(b - 6) = 18$

$a - 6 = 1, b - 6 = 18 \Rightarrow a = 7, b = 24, \Delta = 84$

$a - 6 = 2, b - 6 = 9 \Rightarrow a = 8, b = 15, \Delta = 60$

$a - 6 = 3, b - 6 = 6 \Rightarrow a = 9, b = 12, \Delta = 54.$

8. (90) : $(a - 8)(b - 8) = 32$

$a - 8 = 1, b - 8 = 32, a = 9, b = 40, c = 41, a + b + c = 90$

$a - 8 = 2, b - 8 = 16, a = 10, b = 24, c = 26, a + b + c = 60$

$a - 8 = 4, b - 8 = 8, a = 12, b = 16, c = 20, a + b + c = 48.$

Solution (9-10) :

9. (2) 10. (6)

Since $\lim_{x \to 1} \frac{f(x)}{(x - 1)^2} = 1, \; f(1) = 0$

$$\therefore \quad \lim_{x \to 1} \frac{f(x)}{(x - 1)^2} = \lim_{x \to 1} \frac{f'(x)}{2(x - 1)} = 1$$

$\Rightarrow f'(1) = 0$

$$\therefore \quad \lim_{x \to 1} \frac{f''(x)}{2} = 1 \Rightarrow f''(1) = 2$$

Since $x = 1$ is root of $f(x) = 0$ and $f'(x) = 0$,

$\therefore f(x) = (x - 1)^2(ax^2 + bx + 2) \qquad (\because f(0) = 2)$

$\Rightarrow f'(x) = 2(x - 1)(ax^2 + bx + 2) + (2ax + b)(x - 1)^2$

$\because f'(0) = -6 \Rightarrow b = -2$

Using $f''(1) = 2$, we get $a + b = -1$

$\Rightarrow a = 1 \Rightarrow f(x) = (x - 1)^2 (x^2 - 2x + 2)$

$\Rightarrow f(x) = (x - 1)^4 + (x - 1)^2 \Rightarrow f(2) = 1 + 1 = 2$

Also, $f'(x) = 4(x - 1)^3 + 2(x - 1)$

$\Rightarrow f'(2) = 4 + 2 = 6$

11. (4) : Here $p=\frac{1}{4}, q=1-\frac{1}{4}=\frac{3}{4}$

$\Rightarrow$ 1 – (probability not hitting the target) $> \frac{2}{3}$

$$\Rightarrow 1-{}^nC_0\left(\frac{3}{4}\right)^n > \frac{2}{3} \quad \text{or} \quad \left(\frac{1}{3}\right) > \left(\frac{3}{4}\right)^n$$

$\Rightarrow$ Minimum value of $n = 4$

12. (164) : $p=\frac{1}{{}^6C_1 \times {}^6C_1}=\frac{1}{36}, q=1-p=\frac{35}{36}$

As given

Probability of getting at least one double six $> \frac{99}{100}$

$$\Rightarrow 1-\left(\frac{35}{36}\right)^n > 0.99 \qquad \Rightarrow 0.01 > \left(\frac{35}{36}\right)^n$$

$$\Rightarrow 10^{-2} > \left(\frac{35}{36}\right)^n$$

$$\Rightarrow -2 > n(\log 35 - \log 36) \Rightarrow -2 > -n(0.0122)$$

or $n > \frac{2}{0.0122} = 163.9$

$\therefore \quad n > 163.9 \Rightarrow n = 164$

13. (b) : $V_r = \frac{r}{2}[2r+(r-1)(2r-1)] = \frac{1}{2}[2r^3 - r^2 + r]$

$\therefore \quad V_1 + V_2 + \ldots + V_n$

$$=\frac{1}{2}\left[\frac{2n^2(n+1)^2}{4}-\frac{1}{6}n(n+1)(2n+1)+\frac{n(n+1)}{2}\right]$$

$$= \frac{1}{12}n(n+1)(3n^2+n+2)$$

14. (d) : $T_r = V_{r+1} - V_r - 2$

$$= \frac{1}{2}[2(r+1)^3 - (r+1)^2 + r + 1 - 2r^3 + r^2 - r - 4]$$

$= 3r^2 + 2r - 1 = (r + 1)(3r - 1)$, a composite number.

15. (c) : $\because \lim\limits_{n \to \infty} \frac{1}{n}\left\{\frac{1}{n+1}+\frac{2}{n+2}+\ldots+\frac{3n}{4n}\right\}$

$$= \lim_{n \to \infty} \frac{1}{n}\left\{\frac{1}{n+1}+\frac{2}{n+2}+\ldots+\frac{3n}{n+3n}\right\}$$

$$= \lim_{n \to \infty} \sum_{r=1}^{3n} \frac{1}{n}\left(\frac{r}{n+r}\right) = \lim_{n \to \infty} \sum_{r=1}^{3n} \frac{1}{n}\cdot\left(\frac{\frac{r}{n}}{1+\frac{r}{n}}\right)$$

$$= \int_0^3 \frac{x}{1+x}dx = \int_0^3 \left(1-\frac{1}{1+x}\right)dx$$

$= [x - \ln(1 + x)]_0^3 = 3 - \ln 4 = 3 - 2\ln 2$

16. (b) : Let $P = \lim\limits_{n \to \infty}\left\{\prod_{r=1}^{n}\left(1+\frac{r}{n}\right)\right\}^{1/n}$

$$\therefore \quad \ln P = \lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{n}\ln\left(1+\frac{r}{n}\right) = \int_0^1 \ln(1+x)\,dx$$

$$= [x\ln(1+x)]_0^1 - \int_0^1 \frac{x}{1+x}dx = \ln 2 - \int_0^1 \left(1-\frac{1}{1+x}\right)dx$$

$= \ln 2 - \{x - \ln(1 + x)\}_0^1 = \ln 2 - \{1 - \ln 2\}$

$= 2\ln 2 - 1 = \ln 4 - \ln e = \ln\left(\frac{4}{e}\right)$

$\therefore \; P = \frac{4}{e}$

◆◆

PUBLIC NOTICE JEE ADVANCED 2022 SCHEDULE

S. No.	Activity	Day, Date and Time (IST)
1.	Registration for JEE (Advanced) 2022	Wednesday, June 08, 2022 (10:00 IST) to Tuesday, June 14, 2022 (17:00 IST)
2.	Last date for fee payment of registered candidates	Wednesday, June 15, 2022 (17:00 IST)
3.	Admit Card available for downloading	Monday, June 27, 2022 (10:00 IST) to Sunday, July 03, 2022 (14:30 IST)
4.	Choosing of scribe for PwD candidates	Saturday, July 02, 2022
5.	**JEE (Advanced) 2022**	**Sunday, July 03, 2022** **Paper 1: 09:00-12:00 IST** **Paper 2: 14:30-17:30 IST**
6.	Copy of candidate responses to be available on the JEE (Advanced) 2022 website	Thursday, July 07, 2022 (10:00 IST)
7.	Online display of provisional answer keys	Saturday, July 09, 2022 (10:00 IST)
8.	Feedback and comments on provisional answer keys from the candidates	Saturday, July 09, 2022 (10:00 IST) to Sunday, July 10, 2022 (17:00 IST)
9.	Online declaration of final answer keys	Monday, July 18, 2022 (10:00 IST)
10.	**Result of JEE (Advanced) 2022**	**Monday, July 18, 2022 (10:00 IST)**
11.	Online registration for Architecture Aptitude Test (AAT) 2022	Monday, July 18, 2022 (10:00 IST) to Tuesday, July 19, 2022 (17:00 IST)
12.	Tentative Start of Joint Seat Allocation (JoSAA) 2022 Process	Tuesday, July 19, 2022
13.	Architecture Aptitude Test (AAT) 2022	Thursday, July 21, 2022 (09:00-12:00 IST)
14.	Declaration of results of AAT 2022	Sunday, July 24, 2022 (17:00 IST)

CONCEPT MAP

SETS

Class XI

Representation of Sets

- **Set Builder Form :** A set is described by characterising property $P(x)$ of its element x.
- **Roster or Tabular Form :** A set is described by listing all the elements separated by commas.

Cardinal Properties

If A, B and C are finite sets, U be the finite universal set, then

- $n(A - B) = n(A) - n(A \cap B)$
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B) = n(A) + n(B)$, if A, B are disjoint.
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
- $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$
- $n(\text{exactly two}) = n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$
- $n(\text{exactly one}) = n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(C \cap A) + 3n(A \cap B \cap C)$

SETS

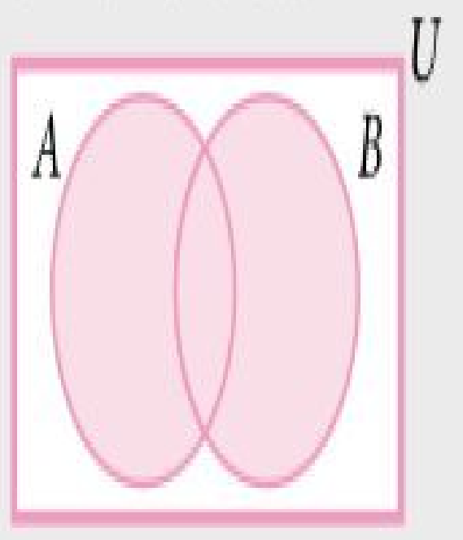
Collection of well defined objects

Types of Sets

- **Singleton set :** A set with only one element.
- **Empty set :** A set with no element.
- **Finite set :** A set contains finite number of elements.
- **Infinite set :** Set which is not finite.
- **Equal sets :** Two sets A and B are equal if they have exactly same elements.
- **Equivalent sets :** Two finite sets A and B are equivalent if they have same number of elements.
- **Universal set :** The set that contains all the sets in the given context.
- **Subset :** A set A is said to be subset of set B if each element of set A is also an element of set B *i.e.*, $A \subseteq B$.
- **Proper subset :** If set A is a subset of set B but $A \neq B$, then $A \subset B$.
- **Superset :** If set A is a subset of set B *i.e.*, $A \subseteq B$, then set B is called superset of set A.
- **Power set :** Set of all subsets of a given set.

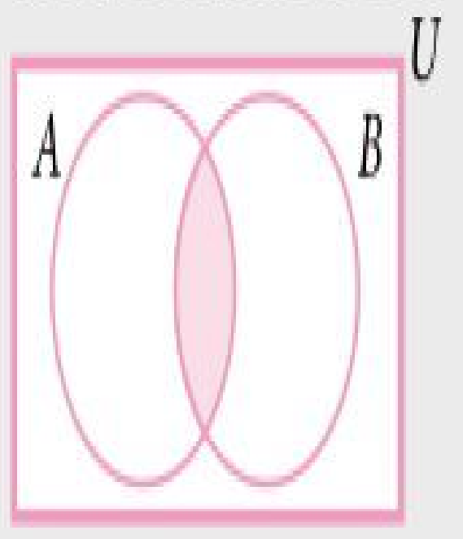
Venn Diagrams

- **Union of Sets**



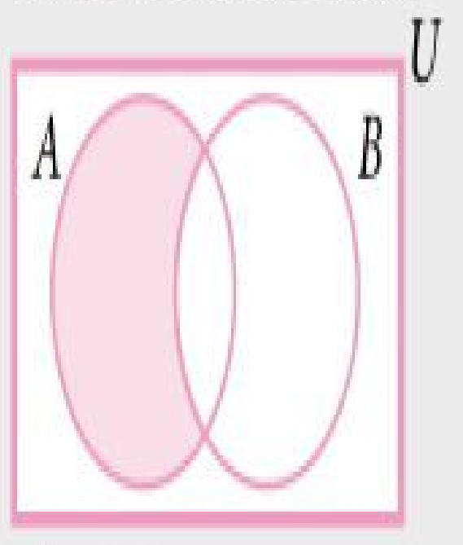
Shaded part represents $A \cup B$

- **Intersection of Sets**



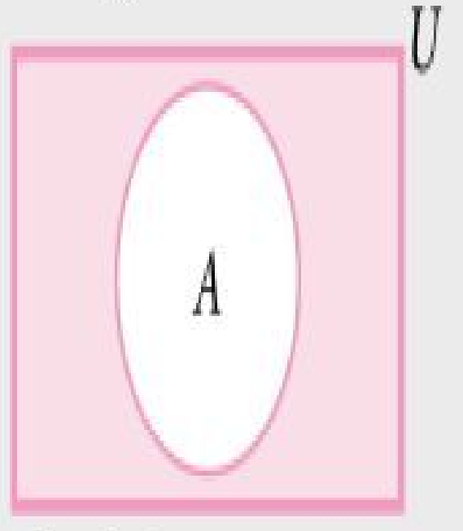
Shaded part represents $A \cap B$

- **Difference of Sets**



Shaded part represents $A - B$

- **Complement of Sets**



Shaded part represents A'

Operations on Sets

Laws of Algebra of Sets

- **Idempotent :** $A \cup A = A$; $A \cap A = A$
- **Commutative :** $A \cup B = B \cup A$; $A \cap B = B \cap A$
- **Associative :** $A \cup (B \cup C) = (A \cup B) \cup C$; $A \cap (B \cap C) = (A \cap B) \cap C$
- **Identity :** $A \cup \phi = \phi \cup A = A$; $A \cap \phi = \phi$; $A \cap U = U \cap A = A$; $A \cup U = U$
- **Distributive :** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

- **Complement Law :** $A \cup A' = U$; $A \cap A' = \phi$
- **De Morgan's Law :** $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$
- **Law of Double Complement :** $(A')' = A$
- **Law of Empty Set and Universal Set :** $\phi' = U$ and $U' = \phi$

Union : It consists of all the elements of set A and set B with common element taken once *i.e.*, $(A \cup B) = \{x : x \in A \text{ or } x \in B\}$

Intersection : It consists of those elements which belong to both set A and B *i.e.*, $(A \cap B) = \{x : x \in A \text{ and } x \in B\}$

Complement : Set of all elements of universal set U which are not the elements of set A *i.e.*, $A' = U - A$.

Difference : Set of elements which belong to set A but not to set B *i.e.*, $A - B = \{x : x \in A, x \notin B\}$

CONCEPT MAP

DETERMINANTS

Class XII

DETERMINANT

Corresponding to every square matrix A, there exists a number called the determinant of A and denoted by $|A|$.

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then $|A| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$.

Properties of Determinants

- The value of a determinant remains unaltered if its rows and columns are interchanged.
- If two rows (or columns) of a determinant are interchanged, then the value of the determinant is multiplied by –1.
- If any two rows (or columns) of a determinant are identical, then the value of determinant is zero.
- If the elements of a row (or column) of a determinant are multiplied by any scalar, then the value of the new determinant is equal to same scalar times the value of the original determinant.
- If each element of any row (or column) of a determinant is the sum of two numbers, then the determinant is expressible as the sum of two determinants of the same order.

Minors and Cofactors

- For any matrix $A = [a_{ij}]_{n \times n}$, if we leave the row and the column of the element a_{ij}, then the value of determinant thus obtained is called the minor of element a_{ij} and it is denoted by M_{ij}.
- The minor M_{ij} multiplied by $(-1)^{i+j}$ is called the cofactor of the element a_{ij} and it is denoted by C_{ij}. *i.e.*, $C_{ij} = (-1)^{i+j} M_{ij}$

Adjoint of a Matrix

Adjoint of a matrix A, denoted by adj (A), is defined as the transpose of the cofactors matrix of A.

Properties : If A is a non-singular matrix of order n, then

- $A(\text{adj } A) = (\text{adj } A)A = |A| I_n$
- $|A(\text{adj}) A| = |A|^n$
- $\text{adj}(AB) = (\text{adj } B) \cdot (\text{adj } A)$
- $|\text{adj } A| = |A|^{n-1}$
- $\text{adj}(\text{adj } A) = |A|^{n-2} A$
- $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$

Area of a Triangle

- Let ABC be a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then area of ΔABC is

$$\Delta = \frac{1}{2}\left\| \begin{matrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{matrix} \right\|$$

- The area of the triangle formed by three collinear points is zero.

Singular and Non-Singular Matrices

A square matrix A of order n is said to be

- **Singular** if $|A| = 0$
- **Non-singular** if $|A| \neq 0$
- If A and B are non-singular matrices of the same order, then AB and BA are also non-singular matrices of the same order.

Inverse of a Matrix

For any square matrix A, inverse of A is defined as $A^{-1} = \frac{1}{|A|}(\text{adj } A), |A| \neq 0$

Properties :

- $(A^{-1})^{-1} = A$
- $(A')^{-1} = (A^{-1})'$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

Solution of System of Linear Equations

Let $AX = B$ be the given system of equations:

- If $|A| \neq 0$, the system is consistent and has unique solution.
- If $|A| = 0$ and $(\text{adj } A)B \neq O$, then the system is inconsistent and hence it has no solution.
- If $|A| = 0$ and $(\text{adj } A)B = O$, then the system is consistent and hence it has infinitely many solutions.

beat the TIME TRAP

Duration : 30 minutes

SECTION-I

Only One Option Correct Type

This section contains 10 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

1. Number of six-digit number in which no digit is repeated, which is divisible by 5, odd numbers occur at even place and even numbers occur at odd place, is

(a) 400 (b) 625 (c) 225 (d) 576

2. The value of $\frac{3+6}{4^{100}}+\frac{3+2\times6}{4^{99}}+\frac{3+3\times6}{4^{98}}+\ldots+\frac{3+100\times6}{4}$ is equal to

(a) $\frac{1}{3}\left(601-\left(\frac{1}{4}\right)^{100}\right)$ (b) $\frac{1}{3}\left(600-\left(\frac{1}{4}\right)^{100}\right)$

(c) $\frac{1}{3}\left(700-\left(\frac{1}{4}\right)^{99}\right)$ (d) $\frac{1}{3}\left(700-\left(\frac{1}{4}\right)^{100}\right)$

3. The family of lines

$$(2\cos\theta+3\sin\theta)x+(3\cos\theta-5\sin\theta)y-(5\cos\theta-2\sin\theta)=0$$

always passes through the point (α, β) $\forall$ $\theta \in R$ then $\alpha+\beta$ is equal to

(a) 1 (b) –1 (c) 2 (d) 0

4. The plane $x-2y+3z=0$ is rotated through a right angle about its line of intersection with the plane $2x+3y-4z-5=0$. The equation of plane in its new position is

(a) $x+3y-5z=17$ (b) $11x+3y-z=10$

(c) $3x+4y-11z=12$ (d) $22x+5y-4z=35$

5. $\int\frac{3x^2+4x-1}{(x^2+1)^2\sqrt{x+1}}dx$ is equal to

(a) $\frac{\sqrt{x+1}}{x^2+1}+c$ (b) $-\frac{2\sqrt{x+1}}{x^2+1}+c$

(c) $\frac{2}{(x^2+1)\sqrt{x+1}}+c$ (d) $-\frac{x}{(x^2+1)\sqrt{x+1}}+c$

6. The general solution of the differential equation $y\{xy-(x^2-y^2)^2\}dx=\{y^3-x(x^2-y^2)^2\}dy$ is equal to

(a) $\frac{y}{x}+\frac{1}{x^2-y^2}=c$ (b) $\frac{x}{y}-\frac{1}{2}\ln\left|\frac{x-y}{x+y}\right|=c$

(c) $\frac{2x}{y}+\frac{1}{x^2-y^2}=c$ (d) $\frac{2x}{y}-\frac{1}{x^2-y^2}=c$

7. A line is drawn at an angle θ with positive direction of x-axis. It intersects the parabola $y^2=8x$ at A and B. If AB is a normal chord which subtend a right angle at the vertex of the parabola, then $\tan^2\theta$ equals

(a) 1/2 (b) 3/4 (c) 1 (d) 2

8. If $f(x)=\frac{3^{x-3}}{3^{1-x}+3^x}$ for all $x \in Q$, then the value of

$$f\left(\frac{1}{109}\right)+f\left(\frac{2}{109}\right)+f\left(\frac{3}{109}\right)+\ldots+f\left(\frac{108}{109}\right)$$ is

(a) 54 (b) 55 (c) 108 (d) 2

9. The value of $\cot\sum\limits_{n=1}^{\infty}\cot^{-1}\left(\frac{2}{n^2}\sum\limits_{k=1}^{n}k^3\right)$ is

(a) 1/2 (b) 1/4 (c) 1/3 (d) 1/6

10. The graph of the function $y=f(x)$ has a unique tangent at $(\pi^3, 0)$. If $A=\lim\limits_{x\to\pi^3}\frac{\ln(1+9f(x))-\sin f(x)}{2f(x)}$, then $\sum\limits_{n=1}^{\infty}A^{-n}$ is equal to

(a) 1/3 (b) 1/2 (c) 4 (d) 1/4

SECTION-II

One or More than One Option(s) Correct Type

This section contains 5 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONE OR MORE are correct.

11. If the intercept made by circle $x^2 + y^2 + x + 5y + 4 = 0$ on lines $x + y + 1 = 0$ and $y = m(x + 1) - 1$ be equal then the values of m can be

(a) 1 (b) –1 (c) –1/7 (d) 7

12. If (0, 0) is one of the vertices and $x - y - 2 = 0$ is one of the sides of an equilateral triangle, then

(a) area of triangle is $\frac{2}{\sqrt{3}}$ sq. unit

(b) orthocentre is $\left(\frac{2}{3}, -\frac{2}{3}\right)$

(c) slope of other two sides of triangle are $-(2+\sqrt{3})$ and $(\sqrt{3}-2)$

(d) circumcentre is $\left(\frac{1}{3}, -\frac{1}{3}\right)$

13. A normal, drawn at a point $P(x, y)$ of a curve meets the x-axis at A and y-axis at B such that $\frac{2}{OA}+\frac{3}{OB}=\frac{1}{5}$, where O is origin. If the curve passes through (7, 11) then

(a) the differential equation of the curve is $(x - 10)dx + (y - 15)dy = 0$

(b) the differential equation of the curve is $(x - 10)dx - (y - 15)dy = 0$

(c) curve passes through (13, 11)

(d) curve passes through (7, 19)

14. Let z_1 and z_2 be the roots of the equations

$$z^2 - 2z\cos\frac{2\pi}{7} + 1 = 0 \text{ and}$$

$$z^2 - 2z\cos\frac{3\pi}{7} + 1 = 0 \quad \text{respectively.}$$

Then which of the following is/are true?

(a) $z_1^2 z_2 + \frac{1}{z_1^2 z_2} = 2$ (b) $z_1^2 z_2 + \frac{1}{z_1^2 z_2} = -2$

(c) $\frac{z_1^3}{z_2^2} + \frac{z_2^2}{z_1^3} = 2$ (d) $\frac{z_1^3}{z_2^2} + \frac{z_2^2}{z_1^3} = -2\cos\frac{5\pi}{7}$

15. The function $f(x)$ satisfying $(f(x))^2 - 4f(x)f'(x) + (f'(x))^2 = 0$ is given by

(a) $ke^{(2+\sqrt{3})x}$ (b) $ke^{(2-\sqrt{3})x}$

(c) $ke^{(2+\sqrt{6})x}$ (d) $ke^{(2-\sqrt{6})x}$

SECTION-III

Integer Value Correct Type

This section contains 5 questions. The answer to each of the question is a single digit integer, ranging from 0 to 9.

16. The maximum value of the function

$$g(x) = \frac{106}{3x^4 + 8x^3 - 18x^2 + 60}, (x > 0) \text{ is } \_\_\_\_\_\_\_.$$

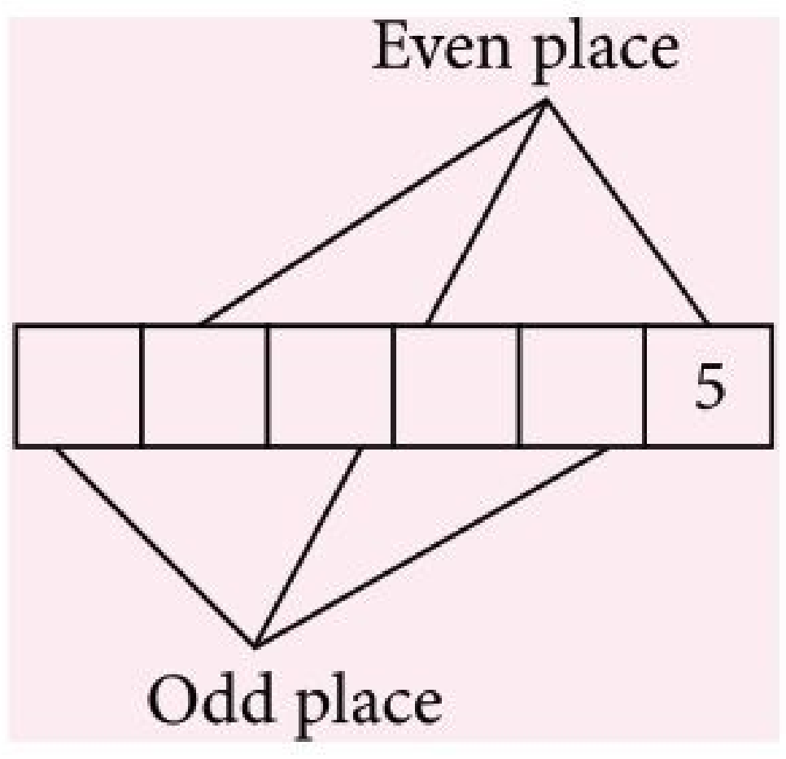
17. Let $p(x) = 2 + 4x + 3x^2 + 5x^3 + 3x^4 + 4x^5 + 2x^6$ for k with $0 < k < 5$, define $I_k = \int_0^\infty \frac{x^k}{p(x)}dx$, the value of k for which I_k is smallest is _______.

18. The number of points where $f(x) = |2 - |2 - |x|||$ is not differentiable is _______.

19. If the sum of first n terms of the A.P. 85, 90, 95, ... is equal to the sum of first $3n$ terms of the A.P. 9, 11, 13, ..., then n equals _______.

20. A nine digit number is formed from 1, 2, 3, 4, 5 such that product of all digits is always 1920. The total number of ways is $393(^9P_r)$, where the value of r is

SOLUTIONS



1. (d) : Even digits = 0, 2, 4, 6, 8

Odd digits = 1, 3, 5, 7, 9

Required no. = 4 × 4 × 4 × 3 × 3

= 576

2. (a) :

$$\underbrace{3\left(\frac{1}{4}+\frac{1}{4^2}+\dots+\frac{1}{4^{100}}\right)}_{S_1}+\underbrace{6\left(\frac{1}{4^{100}}+\frac{2}{4^{99}}+\dots+\frac{100}{4}\right)}_{S_2}$$

$$\text{Now, } S_1 = \frac{3}{4}\left(\frac{1-\left(\frac{1}{4}\right)^{100}}{1-\frac{1}{4}}\right) = 1-\left(\frac{1}{4}\right)^{100}$$

$$\text{and } S_2 = \frac{1}{4^{100}}+\frac{2}{4^{99}}+\dots+\frac{100}{4}$$

$$\text{or } \quad 4S_2 = \frac{1}{4^{99}}+\dots+100$$

$$-3S_2 = \frac{1}{4^{100}}+\left(\frac{1}{4^{99}}+\frac{1}{4^{98}}+\dots+\frac{1}{4}\right)-100$$

$$\Rightarrow \quad S_2 = \frac{100}{3}-\frac{1}{9}\left(1-\left(\frac{1}{4}\right)^{100}\right)$$

$$\therefore\ S=\left(1-\left(\frac{1}{4}\right)^{100}\right)+6\left(\frac{100}{3}-\frac{1}{9}\left(1-\left(\frac{1}{4}\right)^{100}\right)\right)$$

$$=\frac{1}{3}\left(601-\left(\frac{1}{4}\right)^{100}\right)$$

3. (c) : $\cos\theta(2x + 3y - 5) + \sin\theta\,(3x - 5y + 2) = 0$

Required point is intersecting point of

$2x + 3y - 5 = 0$ and $3x - 5y + 2 = 0$

$\Rightarrow \alpha = 1, \beta = 1 \quad \therefore \alpha + \beta = 2$

4. (d) : Equation of plane through the line of intersection of $x - 2y + 3z = 0$ and $2x + 3y - 4z - 5 = 0$ is

$x - 2y + 3z + \lambda(2x + 3y - 4z - 5) = 0 \qquad ....(i)$

(i) is perpendicular to $x - 2y + 3z = 0$

$\therefore\ 1(1 + 2\lambda) - 2(3\lambda - 2) + 3(3 - 4\lambda) = 0$

$\Rightarrow \lambda = 7/8$

Required equation of plane is $22x + 5y - 4z = 35$

5. (b) : $I=\int\frac{3x^2+4x-1}{(x^2+1)^2\sqrt{x+1}}dx=\int\frac{4x^2+4x-x^2-1}{(x^2+1)^2\sqrt{x+1}}dx$

$$=4\int\frac{x(x+1)}{(x^2+1)^2\sqrt{x+1}}dx-\int\frac{dx}{(x^2+1)(\sqrt{x+1})}dx$$

$$=4\int\frac{x(x+1)}{(x^2+1)^2\sqrt{x+1}}dx-$$

$$\left\{\frac{1}{(x^2+1)}2\sqrt{x+1}-\int\frac{-2x}{(x^2+1)^2}2\sqrt{x+1}\,dx\right\}$$

$$=-\frac{2\sqrt{x+1}}{x^2+1}+c$$

6. (c) : $xy^2dx - y(x^2 - y^2)^2dx = y^3dy - x(x^2 - y^2)^2dy$

$\Rightarrow y^2(xdx - ydy) = (x^2 - y^2)^2(ydx - xdy)$

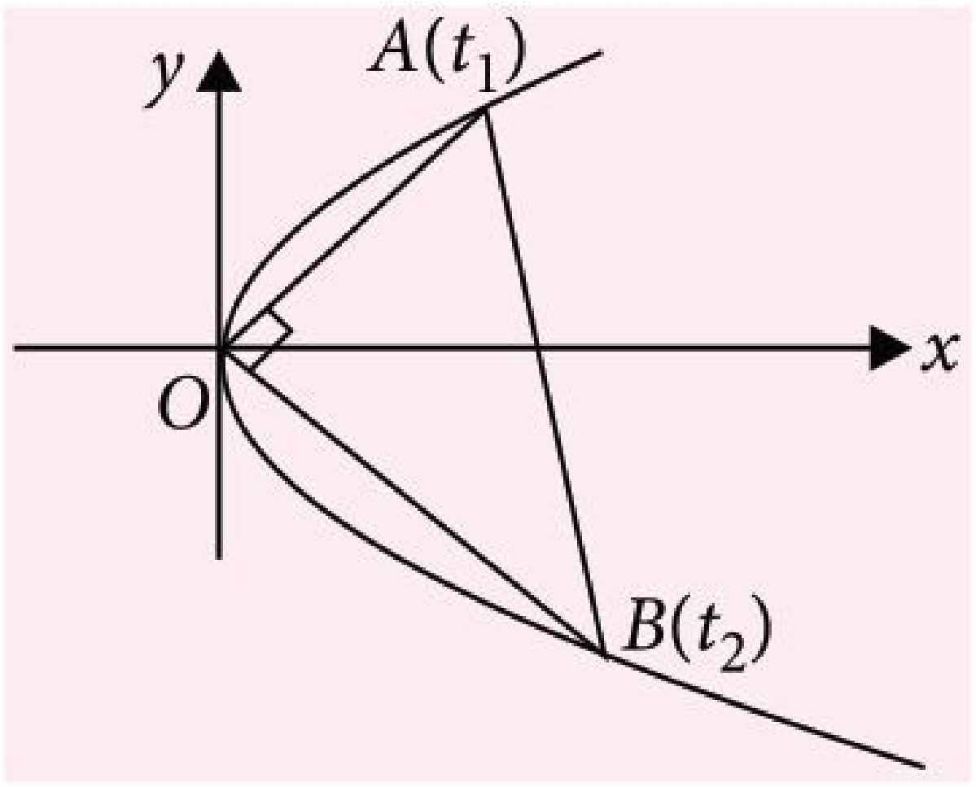
$$\Rightarrow\ \frac{xdx-ydy}{(x^2-y^2)^2}=\frac{ydx-xdy}{y^2}\ \Rightarrow\ \frac{d(x^2-y^2)}{(x^2-y^2)^2}=2d\left(\frac{x}{y}\right)$$

Integrating, we get

$$\frac{2x}{y}+\frac{1}{(x^2-y^2)}=c$$

7. (d) : AB is a normal chord

$\therefore\ t_2=-t_1-\frac{2}{t_1} \qquad ...(i)$



Also, AB subtends right angle at the vertex

$\therefore\ t_1t_2=-4 \Rightarrow t_2=\frac{-4}{t_1} \qquad ...(ii)$

From (i) and (ii), we have

$-\frac{4}{t_1}=-t_1-\frac{2}{t_1}\ \Rightarrow\ t_1^2=2 \ ...(iii)$

Equation of AB is $y + t_1x = 2at_1 + at_1^3$

$\therefore$ The slope of $AB = -t_1 = \tan\theta$

$\Rightarrow \tan^2\theta = t_1^2 = 2$ (from (iii))

8. (d) : $f(x)+f(1-x)=\frac{1}{27}$

$\Rightarrow$ Required sum $=\frac{1}{27}\times 54=2$

9. (c) : $\cot\sum_{n=1}^{\infty}\cot^{-1}\frac{1}{n^2}\times\frac{n^2(n+1)^2}{2}$

$$=\cot\sum_{n=1}^{\infty}\tan^{-1}\frac{2}{(n+1)^2}=\cot\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{n+2-n}{1+n(n+2)}\right)$$

$$=\cot\underbrace{\sum_{n=1}^{\infty}\tan^{-1}(n+2)-\tan^{-1}n}_{S_n}$$

$$S_n=\begin{bmatrix}\tan^{-1}3-\tan^{-1}1\\ \tan^{-1}4-\tan^{-1}2\\ \tan^{-1}5-\tan^{-1}3\\ \vdots\\ \tan^{-1}(n+2)-\tan^{-1}(n)\end{bmatrix}$$

$S_n = \tan^{-1}(n + 2) + \tan^{-1}(n + 1) - \tan^{-1} 2 - \tan^{-1}1$

$$S_n=\tan^{-1}\left(\frac{3n^2+7n}{n^2+9n+10}\right),\ S_\infty=\tan^{-1}3$$

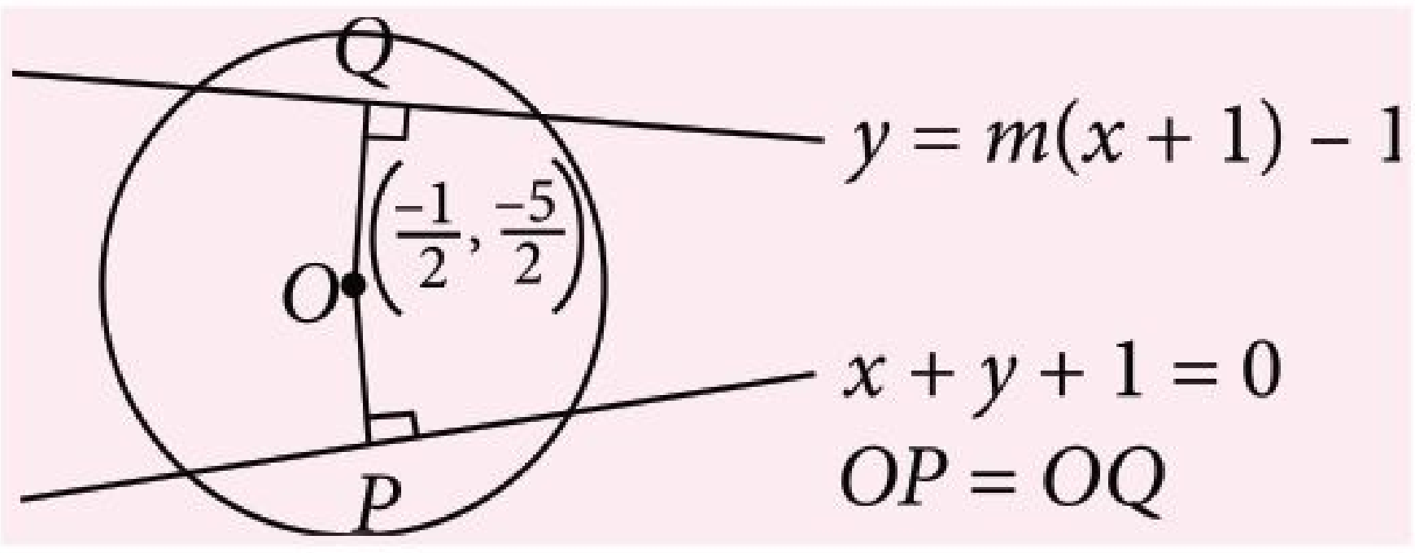
Then $\cot\tan^{-1}3=\cot\ \cot^{-1}\left(\frac{1}{3}\right)=\frac{1}{3}$

10 (a) : $\lim_{x\to\pi^3}\frac{\left\{\frac{1}{1+9f(x)}\right\}9f'(x)-\cos f(x)\cdot f'(x)}{2f'(x)}=\frac{9-1}{2}=4$

$$\sum_{n=1}^{\infty}4^{-n}=\frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}+........\infty=\frac{\frac{1}{4}}{1-\frac{1}{4}}=\frac{1}{3}$$

11. (a, c) : $\left|\frac{-\frac{1}{2}-\frac{5}{2}+1}{\sqrt{2}}\right|=\left|\frac{\frac{3}{2}+\frac{m}{2}}{\sqrt{1+m^2}}\right|$

$$\Rightarrow\ 2=\frac{\frac{9}{4}+\frac{m^2}{4}+\frac{3m}{2}}{(1+m^2)}$$

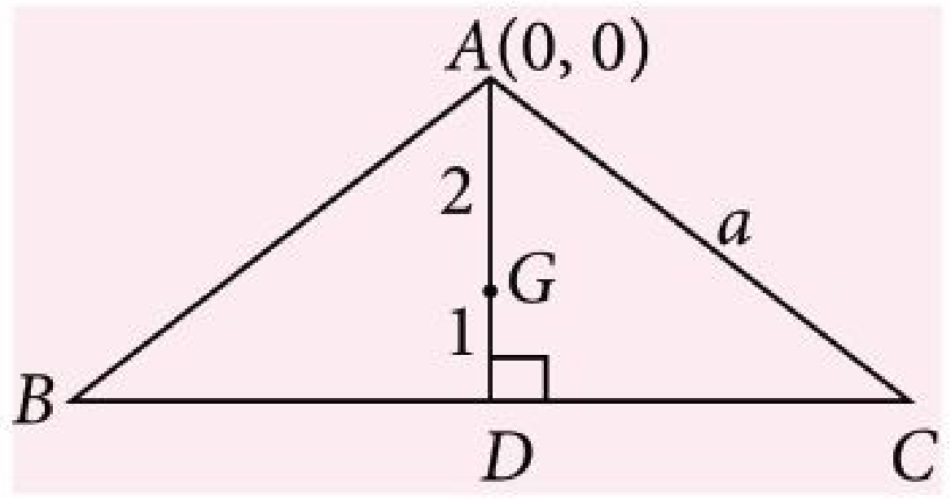


$\Rightarrow\ 2+2m^2=\frac{9}{4}+\frac{m^2}{4}+\frac{3m}{2}\ \Rightarrow\ 7m^2 - 6m - 1 = 0$

$\Rightarrow 7m^2 - 7m + m - 1 = 0 \Rightarrow 7m(m - 1) + 1(m - 1) = 0$

$$\Rightarrow\ m = 1,\ -\frac{1}{7}$$

12. (a, b, c) : Let equation of BC is $x - y - 2 = 0$, Slope of line $x - y - 2 = 0$ is $\tan\theta = 1$



$$AD = \left|\frac{2}{\sqrt{2}}\right| = \sqrt{2} = \frac{\sqrt{3}}{2}a$$

$$\therefore\ a = \frac{2\sqrt{2}}{\sqrt{3}}$$

$$\text{Area} = \frac{\sqrt{3}}{4}\times\frac{8}{3} = \frac{2}{\sqrt{3}}\ \text{sq. units}$$

Equation of AD is $x + y = 0$

$D = (1, -1)$

$$\therefore\ G = \left(\frac{2}{3}, -\frac{2}{3}\right) = \text{orthocenter} = \text{circumcenter}$$

Slopes of other lines are $\tan(\theta \pm 60) = \dfrac{1 \pm \sqrt{3}}{1 \mp \sqrt{3}}$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}}, \frac{1-\sqrt{3}}{1+\sqrt{3}} = -(2+\sqrt{3}), (\sqrt{3}-2)$$

13. (a, c, d) : The equation of normal at P is

$$Y - y = -\frac{1}{\left(\frac{dy}{dx}\right)}(X - x)$$

$$\Rightarrow\ OA = x + yy' \text{ and } OB = \frac{x}{y'} + y$$

$$\therefore\ \frac{2}{OA} + \frac{3}{OB} = \frac{1}{5} \Rightarrow \frac{2}{x+yy'} + \frac{3y'}{x+yy'} = \frac{1}{5}$$

$\Rightarrow$ $10 + 15y' = x + yy' \Rightarrow (y - 15)dy + (x - 10)dx = 0$

$\Rightarrow$ $(x - 10)^2 + (y - 15)^2 = c$, it passes through (7, 11)

$\Rightarrow$ $c = 25$ $\quad\therefore$ $(x - 10)^2 + (y - 15)^2 = 25$

It also passes through (13, 11) and (7, 19).

14. (b, c, d) : $z_1 = \cos\frac{2\pi}{7} \pm i\sin\frac{2\pi}{7} = e^{\frac{2\pi}{7}i}$ or $e^{-\frac{2\pi}{7}i}$

$$z_2 = \cos\frac{3\pi}{7} \pm i\sin\frac{3\pi}{7} = e^{\frac{3\pi}{7}i} \text{ or } e^{\frac{-3\pi}{7}i}$$

$$\therefore\ z_1^2 z_2 + \frac{1}{z_1^2 z_2} = e^{\frac{4\pi}{7}i}\cdot e^{\frac{3\pi}{7}i} + \frac{1}{e^{\frac{4\pi}{7}i}\,e^{\frac{3\pi}{7}i}} = -2$$

$$\frac{z_1^3}{z_2^2} + \frac{z_2^2}{z_1^3} = \frac{e^{\frac{6\pi}{7}i}}{e^{\frac{6\pi}{7}i}} + \frac{e^{\frac{6\pi}{7}i}}{e^{\frac{6\pi}{7}i}} = 2$$

(if z_1 and z_2 are $e^{\frac{2\pi}{7}i}$ and $e^{\frac{3\pi}{7}i}$ respectively)

Also $\dfrac{z_1^3}{z_2^2} + \dfrac{z_2^2}{z_1^3} = \dfrac{e^{\frac{6\pi}{7}i}}{e^{\frac{-6\pi}{7}i}} + \dfrac{e^{\frac{-6\pi}{7}i}}{e^{\frac{6\pi}{7}i}}$

$$= 2\cos\frac{12\pi}{7} = -2\cos\frac{5\pi}{7}$$

15. (a, b) : $\{f(x)\}^2 - 4f(x)\cdot f'(x) + \{f'(x)\}^2 = 0$

$$\Rightarrow\ f'(x) = \frac{4f(x) \pm \sqrt{16(f(x))^2 - 4(f(x))^2}}{2}$$

$$\therefore\ \frac{f'(x)}{f(x)} = 2 \pm \sqrt{3}\quad \therefore\ \log_e f(x) = (2 \pm \sqrt{3})x + c$$

$$\Rightarrow\ f(x) = k\,e^{(2\pm\sqrt{3})x}$$

16. (2) : Let $f(x) = 3x^4 + 8x^3 - 18x^2 + 60$

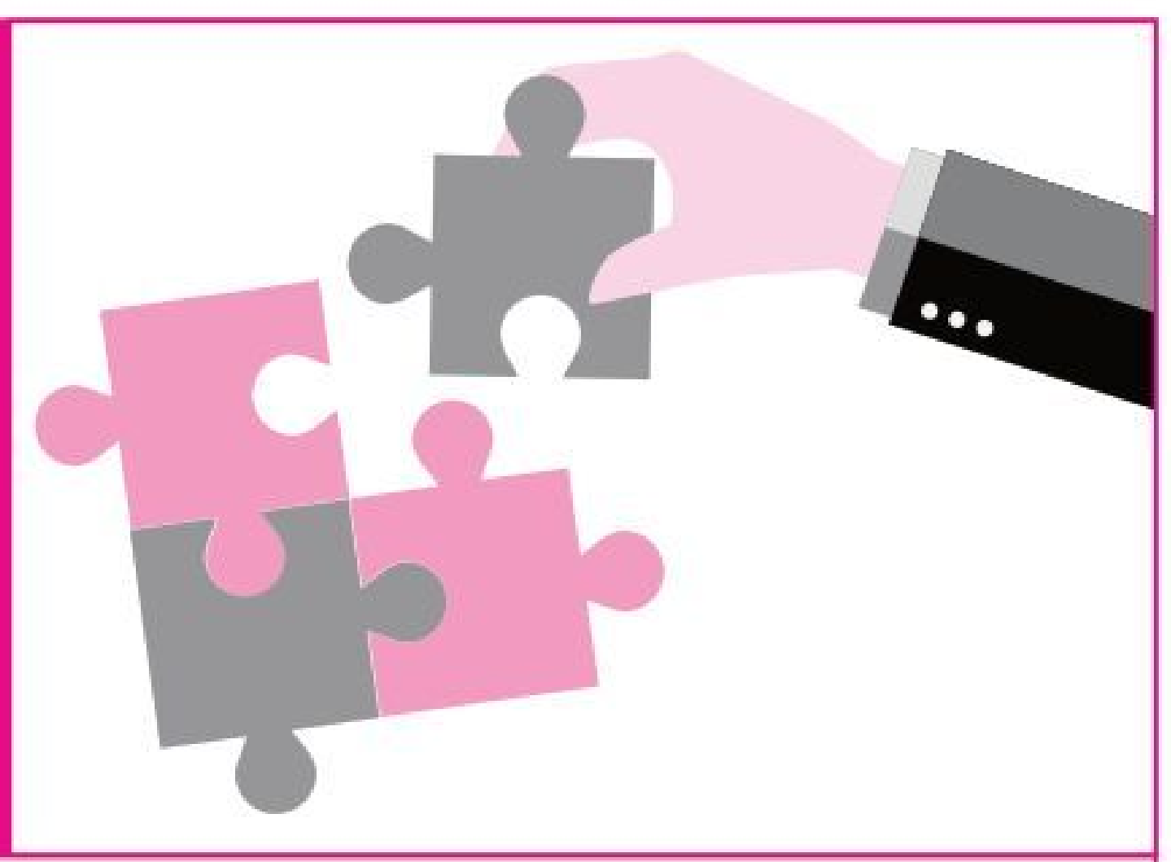
$f'(x) = 12x^3 + 24x^2 - 36x$

$= 12(x^3 + 2x^2 - 3x) = 12x(x^2 + 2x - 3)$

$= 12x(x + 3)(x - 1)$

Also sign of $f'(x)$

PUZZLE CORNER

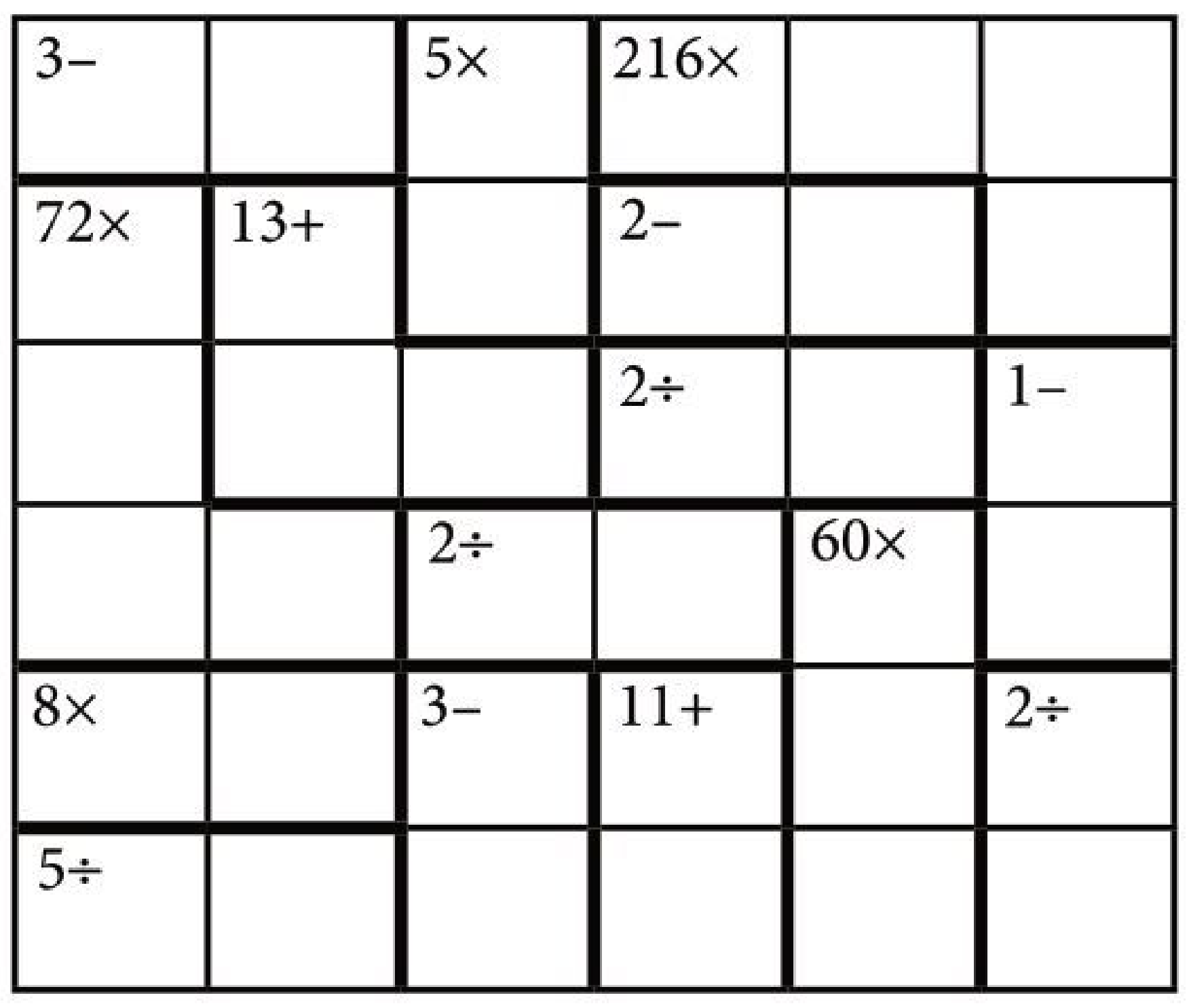


MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics.

In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is multiple of its numbers. For example, if that value is 3 for a two-box cluster, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



3−		5×	216×		
72×	13+		2−		
			2÷		1−
		2÷		60×	
8×		3−	11+		2÷
5÷					

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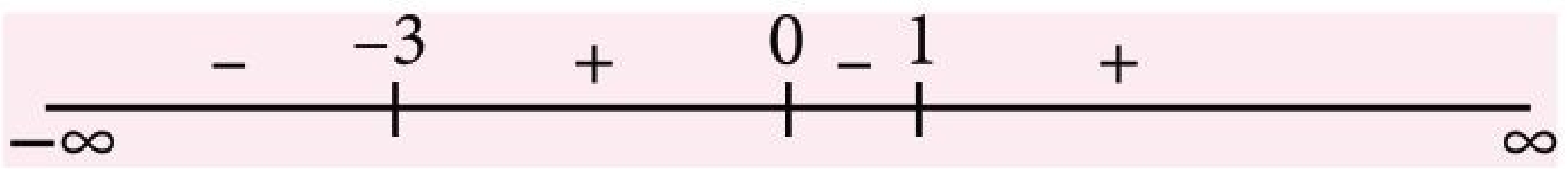


$x \neq -3$ and $x = 1$ is point of local minima.
Now, $f(1) = 3 + 8 - 18 + 60 = 53$

$$\therefore \quad g(x)_{\max} = \frac{106}{53} = 2$$

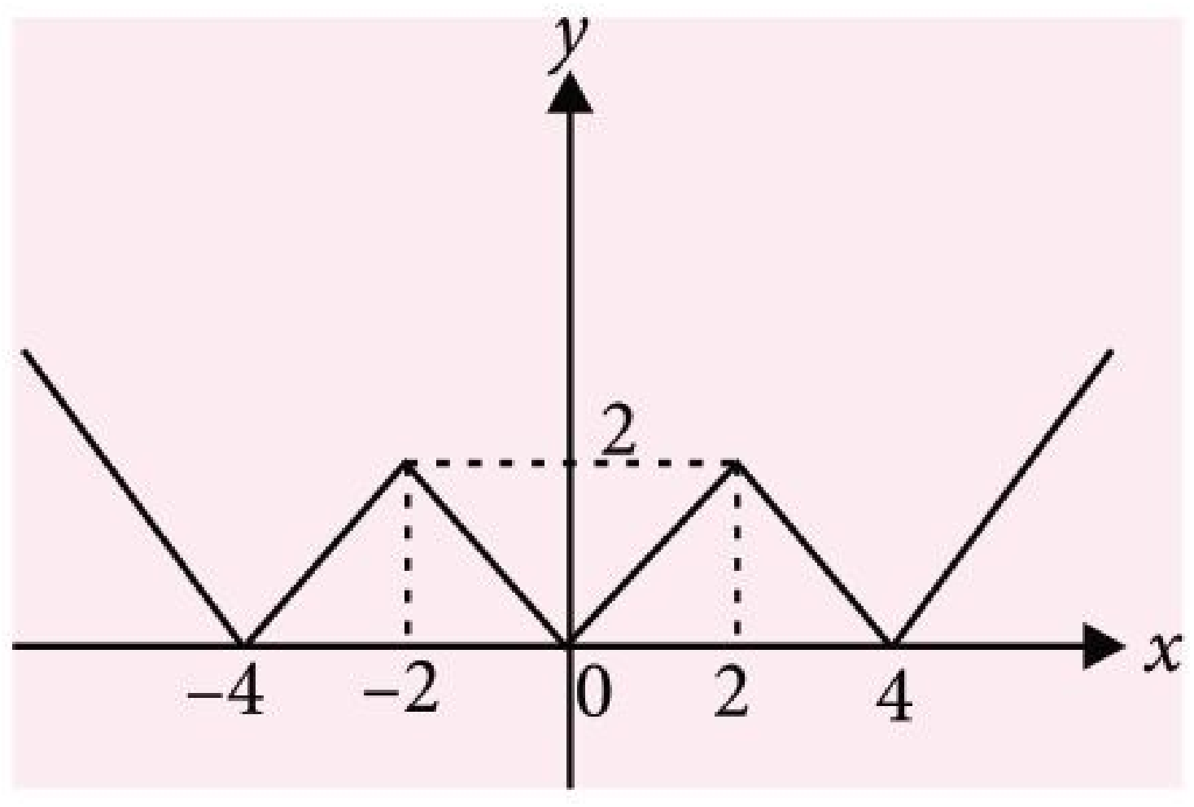
17. (2) : $l_k = \int_{\infty}^{0} \frac{t^{-k}}{t^{-6}p(t)}\left(\frac{-dt}{t^2}\right) = l_{4-k}$

Now, $l_k = \frac{(l_k + l_{4-k})}{2} = \int_0^{\infty} \frac{(x^k + x^{4-k})}{2p(x)}dx \geq \int_0^{\infty} \frac{x^2 dx}{p(x)} = l_2$

$$\left(\frac{x^k + x^{4-k}}{2} \geq x^2 \text{ (AM-GM inequality)}\right)$$

$\therefore \quad l_k$ is smallest for $k = 2$.

18. (5) : As shown in the figure, $f(x)$ is non differentiable at 5 points.



19. (9) : $\frac{n}{2}[2\times 85+(n-1)\times 5] = \frac{3n}{2}[2\times 9+(3n-1)\times 2]$

$\Rightarrow \quad 5n + 165 = 48 + 18n$

$\Rightarrow \quad 13n = 117 \quad \Rightarrow \quad n = 9$

20. (2) : $1920 = 5 \times 3 \times 2^7$

$\Rightarrow \quad 1920 = 5 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$$= \frac{9!}{7!} = 9 \times 8$$

$\Rightarrow \quad 1920 = 5 \times 3 \times 4 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1$

$$= \frac{9!}{5!} = 9 \times 8 \times 7 \times 6$$

3, 5, 4, 4, 2, 2, 2, 1, 1

$$\Rightarrow \quad \frac{9!}{2!2!3!}$$

3, 5, 4, 4, 4, 2, 1, 1, 1

$$\Rightarrow \quad \frac{9!}{3!3!}$$

$$\therefore \quad \text{Total} = \frac{9!}{7!} + \frac{9!}{5!} + \frac{9!}{2!2!3!} + \frac{9!}{3!3!}$$

$$= \frac{9!}{7!}[1 + 42 + 210 + 140] = 393 \cdot {}^9P_2$$

◆

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JEE WORK OUTS

PAPER-1

SECTION-I

SINGLE CORRECT OPTION

This section contains 8 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

1. On a railway there are 10 stations. The number of types of tickets required in order that it may be possible to book a passenger from every station to every other is

(a) $\frac{10!}{8!}$ (b) $10!\,2!$ (c) $\frac{10!}{2!}$ (d) $\frac{10!}{8!\,2!}$

2. If the roots of $ax^2 + bx + c = 0$ are of the form $\frac{m}{m-1}$ and $\frac{m+1}{m}$ then the value of $(a + b + c)^2$ is

(a) $b^2 - 2ac$ (b) $2b^2 - ac$
(c) $b^2 - 4ac$ (d) $2(b^2 - 2ac)$

3. If A and B are two square matrices of order 3×3 which satisfy $AB = A$ and $BA = B$, then $(A + B)^7$ is

(a) $7(A + B)$ (b) $7 \cdot I_{3 \times 3}$
(c) $64(A + B)$ (d) $128\,(A + B)$

4. If a_1, a_2, a_3, a_4, a_5 are in H.P., then $a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5$ is equal to

(a) $2a_1a_5$ (b) $3a_1a_5$ (c) $8a_1a_5$ (d) $4a_1a_5$

5. The maximum value of $(\sin\alpha_1)(\sin\alpha_2) \ldots (\sin\alpha_n)$ under the restrictions $0 \le \alpha_1, \alpha_2, \ldots \alpha_n < \pi/2$ and $(\tan\alpha_1)(\tan\alpha_2) \ldots. (\tan\alpha_n) = 1$ is

(a) $\frac{1}{2^n}$ (b) $\frac{1}{2n}$ (c) $\frac{1}{2^{n/2}}$ (d) 1

6. In a conference 10 speakers are to give their speaches one after another. Find the probability of the event if S_1 speaks before S_2 and S_2 speaks before S_3 and the remaining 7 speakers have no objection to speak at any number?

(a) 1/6 (b) 1/3 (c) 1/45 (d) 3/10

7. i^i is a

(a) complex number
(b) purely imaginary number
(c) real number (d) none of these

8. If k and K are minimum and maximum values of $\sin^{-1}x + \cos^{-1}x + \tan^{-1}x$ respectively, then

(a) $k = \frac{\pi}{4}, K = \frac{3\pi}{4}$ (b) $k = 0, K = \pi$
(c) $k = \frac{\pi}{2}, K = \pi$ (d) not defined

SECTION-II

MULTIPLE CORRECT OPTION

This section contains 4 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

9. If $f(x)$ and $g(x)$ are functions such that $f(x + y) = f(x) \cdot g(y) + g(x) \cdot f(y)$ then

$$\begin{vmatrix} f(\alpha) & g(\alpha) & f(\alpha+\theta) \\ f(\beta) & g(\beta) & f(\beta+\theta) \\ f(\gamma) & g(\gamma) & f(\gamma+\theta) \end{vmatrix}$$ is independent of

(a) α (b) β (c) γ (d) θ

10. Sum of the roots of the equation $(x + 1) = 2\log_2(2^x + 3) - 2\log_4(1980 - 2^{-x})$ is greater than

(a) 2 (b) 3 (c) 4 (d) 5

11. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$, then

(a) $P(B/A) = \frac{P(B)}{P(A)}$
(b) $P(A^C \cup B^C) = P(A^C) + P(B^C)$
(c) $P((A \cup B)^C) = P(A^C)\,P(B^C)$
(d) $P(A/B) = P(A)$

12. The origin and roots of the equation $z^2 + pz + q = 0$ form an equilateral triangle if

(a) $p^2 = q$ (b) $p^2 = 3q$
(c) $q^2 = 3p$ (d) $q^2 = p$

SECTION-III

LINKED COMPREHENSION TYPE

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which one or more than one can be correct.

Paragraph for Question Nos. 13 to 15

Let p be a prime number and n be a positive integer, then exponent of p is $n!$ is denoted by $E_p(n!)$ and is given by

$$E_P(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots + \left[\frac{n}{p^k}\right]$$

where $p^k < n < p^{k+1}$ and $[x]$ denotes the integral part of x.

If we isolate the power of each prime contained in any number N, then N can be written as

$N = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3}$ .... where p_1, p_2, p_3 ... prime numbers and $\alpha_1, \alpha_2, \alpha_3$, ... are natural numbers.

13. The numbers of zeroes at the end of 108! is

(a) 24 (b) 25 (c) 26 (d) 21

14. The number of prime numbers among the numbers 101! + 2, 101! + 3, 101! + 4, ..., 101! + 101 is

(a) 31 (b) 29
(c) 53 (d) none of these

15. The last non-zero digit in 20! must be equal to

(a) 2 (b) 3 (c) 4 (d) 9

Paragraph for Question Nos. 16 to 18

The numbers 1, 3, 6, 10, 15, 21, 28, …… are called triangular numbers. Let t_n denotes the n^{th} triangular number then it can be observed that $t_1 = 1$, $t_2 = 3$, $t_n = t_{n-1} + n$.

16. t_{100} must be equal to

(a) 5050 (b) 5151
(c) 5252 (d) none of these

17. If m is n^{th} triangular number then

(a) $n = \dfrac{\sqrt{1+8m}+1}{2}$ (b) $n = \dfrac{\sqrt{1+8m}-1}{2}$

(c) $n = \dfrac{\sqrt{1+4m}-1}{2}$ (d) none of these

18. The number of positive integers lying between t_{50} and t_{51} must be

(a) 50 (b) 51
(c) 52 (d) none of these

SECTION-IV

MATRIX-MATCH TYPE

This section contains 2 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II.

19. Match the following:

Column I		Column II	
(A)	If n be the number of ways in which 12 different books can be distributed equally among 3 persons, then $\dfrac{(4!)^4 n}{12!}$ is divisible by	p.	4
(B)	If λ be the number of integral solutions of $x + y + z = 15$ such that $x \geq 1$, $y \geq 2$ and $z \geq 3$ then λ is divisible by	q.	6
(C)	If λ be maximum number of points of intersection of 8 straight lines then λ is divisible by	r.	12
(D)	A car will hold 2 persons in the front seat and 1 in the near seat. If among 6 persons only 2 can drive, the number of ways in which car can be filled is λ then λ is divisible by	s.	5

20. If in a triangle $2a^2 + 4b^2 + c^2 = 4ab + 2ac$, then match the following:

Column I		Column II	
(A)	$\cos A$	p.	1/4
(B)	$\cos B$	q.	7/8
(C)	$\cos C$	r.	0
(D)	$\sin(A - C)$	s.	1

PAPER-2

SECTION-I

SINGLE CORRECT OPTION

This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

1. The remainder when 5^{97} is divided by 52 is
(a) 2 (b) 3 (c) 5 (d) 10

2. The number of real solutions of (x, y), where $y = |\sin x|$, $y = \cos^{-1}(\cos x)$, $-2\pi \le x \le 2\pi$, is
(a) 2 (b) 1 (c) 3 (d) 4

3. In ΔABC, $\angle A = \angle C = \pi/6$ and radius of incircle is r, if radius of circumscribing circle $R = 4$ then r is equal to
(a) $4\sqrt{3}-6$ (b) $4\sqrt{3}+6$
(c) $2\sqrt{3}-2$ (d) $2\sqrt{3}+2$

4. If $2x = -1+\sqrt{3}i$, then the value of
$(1 - x^2 + x)^6 - (1 - x + x^2)^6 =$
(a) 32 (b) −64 (c) 64 (d) 0

SECTION-II

MULTIPLE CORRECT OPTION

This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

5. Let $\{\Delta_1, \Delta_2, \Delta_3, ..., \Delta_k\}$ be the set of third-order determinants that can be made with the distinct non-zero real numbers $a_1, a_2, a_3, ..., a_9$. Then
(a) $k = 9!$ (b) $\sum_{i=1}^{k} \Delta_i = 0$
(c) at least one $\Delta_i = 0$ (d) none of these

6. The quadratic equation $x^2 - 2x - \lambda = 0, \lambda \ne 0,$
(a) cannot have a real roots if $\lambda < -1$
(b) can have a rational root if λ is a perfect square of an integer
(c) cannot have an integral root if $n^2 - 1 < \lambda < n^2 + 2n$ where $n = 0, 1, 2, 3, ...$
(d) none of these

7. Given the set of four real numbers. First three numbers are in G.P. and last three numbers are in A.P. The common difference of A.P. is 6. First and fourth terms are same. According to the given data,
(a) fourth term = 4 (b) fourth term = 8
(c) sum of 4 nos. = 14 (d) sum of 4 nos. = 16

8. If $x + y + z = 5$ and $xy + yz + zx = 3$; $x, y, z \in R$ then for x,
(a) largest value $= \frac{13}{3}$ (b) largest value $= \frac{10}{3}$
(c) least value = −1 (d) least value = 5

9. In the expansion of $\left(\sqrt[3]{4}+\frac{1}{\sqrt[4]{6}}\right)^{20}$,
(a) the number of irrational terms = 19
(b) middle term is irrational
(c) the number of irrational terms = 15
(d) 9^{th} term is rational

SECTION-III

MATRIX-MATCH TYPE

This section contains 2 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II.

10. Match the following:

Column I		Column II	
(A)	Any two small squares on a chess board are chosen at random. Probability that they have a common side is	p.	1/10
(B)	One of ten keys opens the door. If we try the keys one after another, then the probability that the door is opened on the tenth attempt is	q.	0
(C)	The probability of obtaining no head in an infinite sequence of independent tosses of a coin is	r.	$\frac{7}{144}$
(D)	Two squares are chosen at random from the small squares drawn on a chess board. The chance that the two squares chosen have exactly one corner in common is	s.	1/18

11. Match the following:

Column I		Column II	
(A)	M_r is defined as : $M_r = \begin{bmatrix} r-1 & \frac{1}{r} \\ 1 & \frac{1}{(r-1)^2} \end{bmatrix}$ and $\lvert M_r \rvert$ is the corresponding determinant value of M_r, then $\lim_{n\to\infty}(\lvert M_2\rvert+\lvert M_3\rvert+...+\lvert M_n\rvert)=$	p.	0
(B)	If $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$ then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$	q.	4
(C)	If $A = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$ and a matrix C is defined as $C = (BAB^{-1})(B^{-1}A^TB)$, where $\lvert B\rvert \neq 0$, then $\lvert C\rvert = k^2$, $(k \in N)$ where $k =$	r.	2
(D)	If $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ and $A^4 = -\lambda I$, then λ equals	s.	1

SECTION-IV

INTEGER ANSWER TYPE

This section contains 8 questions. The answer to each of the questions is a single–digit integer, ranging from 0 to 9.

12. Find the number of roots of the equation $z^{10} = 1$ satisfying $|\arg z| < (\pi/2)$.

13. Find the number of quadratic equations which remain unchanged by squaring their roots.

14. $A(0, 0)$, $B(4, 2)$ and $C(6, 0)$ are the vertices of a triangle ABC and BD is its altitude. The line through D parallel to the side AB intersects the side BC at a point E. Find the product of areas of ΔABC and ΔBDE.

15. A coin is tossed twice and the four possible outcomes are assumed to be equally likely. If A is the event, 'both head and tail have appeared', and B be the event, 'at most one tail is observed', then $5[P(B/A)]$ is

16. The sum of the roots of the equation $4\sin^3\left(\frac{\pi}{2}+x\right) - 4\cos^2 x - \cos(\pi + x) - 1 = 0$ in the interval $[0, 4\pi]$ is $p\pi$. Find p.

17. In ΔABC, $\angle A = 30°$ and $\angle C = 105°$. Find k such that $k = \left(\frac{c^2-b^2}{a^2}\right)^2$.

18. Find the number of integral values of λ if $(\lambda, 2)$ is an interior point of ΔABC formed by $x + y = 4$, $3x - 7y = 8$, $4x - y = 31$.

19. The equation $ax^2 + bx + 6 = 0$ does not have two distinct real roots, then the least value of $3a + b + 3$ is

ANSWERS

PAPER-1

1. (a) **2.** (c) **3.** (c) **4.** (d) **5.** (c)
6. (a) **7.** (c) **8.** (a) **9.** (a, b, c, d)
10. (a, b) **11.** (c, d) **12.** (b) **13.** (b) **14.** (d)
15. (c) **16.** (a) **17.** (b) **18.** (a)
19. (A) → (p, q, r); (B) → (s); (C) → (p); (D) → (p, s)
20. (A) → (p) ; (B) → (q) ; (C) → (p) ; (D) → (r)

PAPER-2

1. (c) **2.** (c) **3.** (a) **4.** (d) **5.** (a, b)
6. (a, c) **7.** (b, c) **8.** (a, c) **9.** (a, b, d)
10. (A) → (s) ; (B) → (p) ; (C) → (q) ; (D) → (r)
11. (A) → (s) ; (B) → (r) ; (C) → (q) ; (D) → (q)
12. (5) **13.** (4) **14.** (8) **15.** (5) **16.** (6)
17. (3) **18.** (1) **19.** (1)

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10 BEST PROBLEMS
for JEE Advanced

1. Given that

$$iz^2 = 1+\frac{2}{z}+\frac{3}{z^2}+\frac{4}{z^3}+\frac{5}{z^4}+\ldots\ldots \text{ and } z = n\pm\sqrt{-i}.$$

Find the value of n.

2. Find the value of x by evaluating the given series

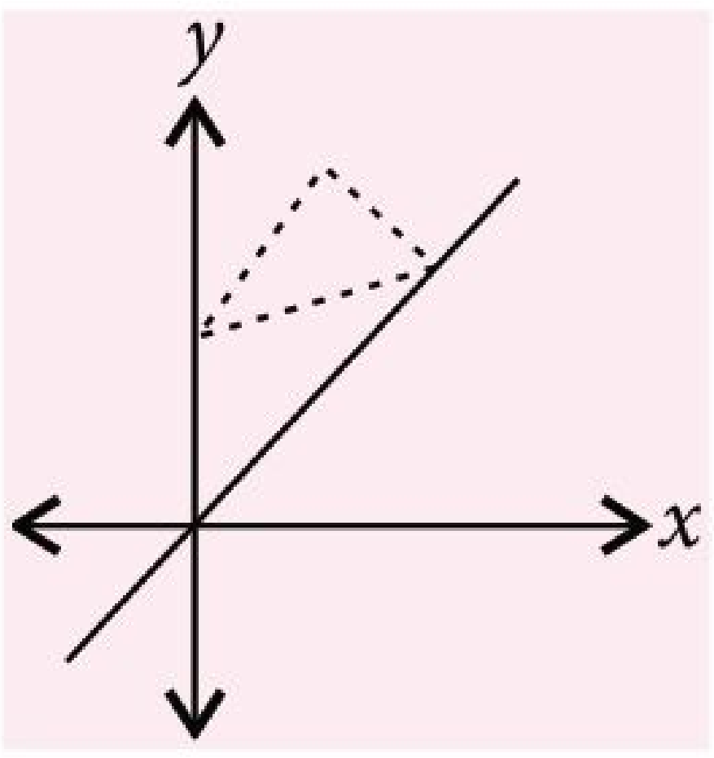
$$1+\frac{1}{5}+\frac{1\times3}{5\times10}+\frac{1\times3\times5}{5\times10\times15}+\ldots\ldots\infty=\sqrt{x},\ x\in Q$$

3. If r_1, r_2, r_3, r_4, r_5 are the complex roots of the equation $x^5 - 3x^4 - 1 = 0$. Find the value of

$$\frac{1}{r_1^9}+\frac{1}{r_2^9}+\frac{1}{r_3^9}+\frac{1}{r_4^9}+\frac{1}{r_5^9}$$

4. Find the sum of all (distinct) complex values of c for which the polynomial $f_c(x) = x^4 - (c^2 - 7c + 11)x^2 + (18 - 21c + 8c^2 - c^3)$ has strictly less than four distinct complex zeroes.

5. The minimum possible perimeter of a triangle with one vertex at (3, 9), one anywhere on the y-axis and one anywhere on the line $y = x$ is



6. $$\lim_{n\to\infty}\sum_{r=3}^{n}\frac{r^3-8}{r^3+8}=$$

7. Inside a square $ABCD$, points P and Q are positioned so that $DP \parallel QB$ and $DP = PQ = QB$. Of all configurations that satisfy these requirements, what is the minimum possible value of $\angle ADP$, (in degrees)?

8. Find the value of $\int_0^{2n\pi}\max\left\{\sin x, \sin^{-1}(\sin x)\right\}dx.$ Draw the graph of the same.

9. Let P be a point (other than the origin) lying on the parabola $y = x^2$. The normal line to the parabola at P will intersect the parabola at another point Q. The minimum possible value for the area bounded by the line PQ and the parabola is

10. Find the value of $\lim_{x\to0}\left(\int_0^1 (by+a(1-y))^x\,dy\right)^{\frac{1}{x}}$ where $b > a$.

SOLUTIONS

1. Multiplying both sides of the equation by z, we get

$$iz^3 = z+2+\frac{3}{z}+\frac{4}{z^2}+\ldots\ldots\ldots \qquad \ldots\text{(i)}$$

and subtracting the original equation from (i) we get

$$iz^2(z-1) = z+1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\ldots\ldots\ldots$$

Using the formula for an infinite geometric series, we find

$$iz^2(z-1)=\frac{z}{1-\frac{1}{z}}=\frac{z^2}{z-1}$$

Rearranging, we get

$$iz^2(z-1)^2 = z^2 \Rightarrow (z-1)^2 = \frac{1}{i} \Rightarrow z = 1\pm\sqrt{-i}.$$

Thus $n = 1$

2. Each term is of the following form :

$$\frac{\prod_{k=1}^{n}(2k-1)}{\prod_{k=1}^{n}(5k)}$$

$$=\frac{\prod_{k=1}^{n}(2k-1)\prod_{k=1}^{n}(2k)}{(n!)5^n\prod_{k=1}^{n}(2k)}=\frac{(2n)!}{(n!)5^n(n!)2^n}=\frac{1}{10^n}\binom{2n}{n}$$

Here $\binom{n}{r} = {}^nC_r$

$$s=\sum_{n=0}^{\infty}\frac{1}{10^n}\binom{2n}{n}$$

Now, since the result is the square root of a rational number, let's find s^2. Using the Cauchy Product (with $\frac{1}{10}$ as the independent variable), we get the following formula.

$$s^2=\sum_{n=0}^{\infty}\frac{1}{10^n}\sum_{k=0}^{n}\binom{2k}{k}\binom{2(n-k)}{n-k}$$

Now it can be shown that for all whole numbers n, we have $\sum_{k=0}^{n}\binom{2k}{k}\binom{2(n-k)}{n-k}=4^n$

Hence, we have

$$s^2=\sum_{n=0}^{\infty}\frac{1}{10^n}\times 4^n=\sum_{n=0}^{\infty}\left(\frac{2}{5}\right)^n=\frac{1}{1-\frac{2}{5}}=\frac{5}{3}$$

Thus, $x=\frac{5}{3}$

3. From the given polynomial, we have

$$\sum r_i=3 \text{ and } \sum r_i^{-1}=0$$

The sum of the reciprocals of the roots come from the fact that the polynomial with reciprocal roots has its coefficients reversed.

$$x^5-3x^4-1=0 \Rightarrow x^4(x-3)=1$$

$$\Rightarrow x^4=(x-3)^{-1} \Rightarrow x^{-9}=\frac{(x-3)^2}{x}$$

$$\sum r_i^{-9}=\sum\frac{(r_i-3)^2}{r_i}=\sum\frac{r_i^2-6r_i+9}{r_i}$$

$$=\sum(r_i-6+9r_i^{-1})=\sum r_i-\sum 6+9\sum r_i^{-1}$$

$$=3-5(6)+0=-27$$

4. The polynomial $f_c(x)$ will fail to have four distinct complex zeros when the quadratic polynomial

$g_c(x)=x^2-(c^2-7c+11)x+(18-21c+8c^2-c^3)$ either has repeated roots or has 0 as a root.

Case 1: One of the roots of $g_c(x)$ is zero precisely when

$$c^3-8c^2+21c-18=(c-2)(c-3)^2=0$$

and so precisely when $c=2, 3$

Case 2 : The roots of $g_c(x)$ are repeated when its discriminant is zero, so that

$$(c^2-7c+11)^2-4(18-21c+8c^2-c^3)=0$$

$$\Rightarrow c^4-10c^3+39c^2-70c+49=0$$

$$\Rightarrow (c^2-5c+7)^2=0$$

and this happens for two distinct values of c which sum to 5.

Thus the sum of the possible values of c is

$$2+3+5=10.$$

5. Let A be the vertex (3,9), B be the vertex on the y-axis and C be the vertex on the line $y=x$. Also, let $D(-3, 9)$ be the reflection of A in the y-axis and $E(9, 3)$ be the reflection of A in the line $y=x$. Then $AB=BD$ and $AC=CE$, and thus the perimeter of ΔABC is equal to $DB+BC+CE$.

But the shortest distance between two points is a straight line, so

$DB+BC+CE \geq DE$

$$=\sqrt{(9-(-3))^2+(3-9)^2}=\sqrt{180}=6\sqrt{5}$$

This minimum can be obtained by then choosing B and C as the points of intersection of the line DE with the y-axis and the line $y=x$, respectively. This gives us the points $B\left(0,\frac{15}{2}\right)$ and $C(5, 5)$. This will yield a perimeter for ΔABC of $6\sqrt{5}$.

6. $$\lim_{n\to\infty}\left(\frac{3^3-8}{3^3+8}\right)\left(\frac{4^3-8}{4^3+8}\right)\cdots\cdots\left(\frac{n^3-8}{n^3+8}\right)$$

$$=\lim_{n\to\infty}\left(\frac{3-2}{3+2}\cdot\frac{3^2+4+2(3)}{3^2+4-2(3)}\right)\left(\frac{4-2}{4+2}\cdot\frac{4^2+4+2(4)}{4^2+4-2(4)}\right)$$

$$\cdots\cdots\left(\frac{n-2}{n+2}\cdot\frac{n^2+4+2(n)}{n^2+4-2(n)}\right)$$

$$=\lim_{n\to\infty}\left\{\frac{3-2}{3+2}\cdot\frac{4-2}{4+2}\cdot\frac{5-2}{5+2}\cdots\cdots\frac{n-2}{n+2}\right\}$$

$$\left\{\frac{3^2+4+2(3)}{3^2+4-2(3)}\cdot\frac{4^2+4+2(4)}{4^2+4-2(4)}\cdots\cdots\frac{n^2+4+2(n)}{n^2+4-2(n)}\right\}$$

$$=\lim_{n\to\infty}\left\{\frac{1\cdot2\cdot3\cdot4\cdot5\cdot6\ldots\ldots}{5\cdot6\cdot7\cdot8\ldots\ldots}\right\}\left\{\frac{19\cdot28\cdot39\cdot52\cdot63\ldots\ldots}{7\cdot12\cdot19\cdot28\cdot39\cdot52\cdot63\ldots\ldots}\right\}$$

$$=\lim_{n\to\infty}(1\cdot2\cdot3\cdot4\cdot)\left(\frac{1}{7\cdot12}\right)=\frac{2}{7}$$

7. Without loss of generality,

let the corners of the square be $A(0,2)$, $B(2,2)$, $C(2,0)$ and $D(0,0)$. Now let point P has coordinates (a, b); then by symmetry the coordinates of point Q must be $(2-a, 2-b)$. Then since $DP=PQ$, we have that

$$a^2+b^2=(2-2a)^2+(2-2b)^2$$

$$\Rightarrow 3a^2+3b^2-8a-8b+8=0$$

$$\Rightarrow \left(a-\frac{4}{3}\right)^2+\left(b-\frac{4}{3}\right)^2=\frac{8}{9}$$

This means that P lies on a circle centered at $O\left(\frac{4}{3},\frac{4}{3}\right)$ with radius $r=\frac{2}{3}\sqrt{2}$.

Therefore, $\angle ADP$ will be minimized when DP is tangent to this circle. Now by symmetry OD makes an angle of 45° with the x-axis and has length $\frac{4}{3}\sqrt{2}$.

Thus, $\angle PDO=45°-\angle ADP$, and so

$$\sin(\angle PDO)=\sin(45°-\angle ADP)=\frac{OP}{OD}=\frac{\frac{2}{3}\sqrt{2}}{\frac{4}{3}\sqrt{2}}=\frac{1}{2}$$

This implies that $45°-\angle ADP=\sin^{-1}\left(\frac{1}{2}\right)=30°$, and

So the minimum value for $\angle ADP$ is $45°-30°=15°$

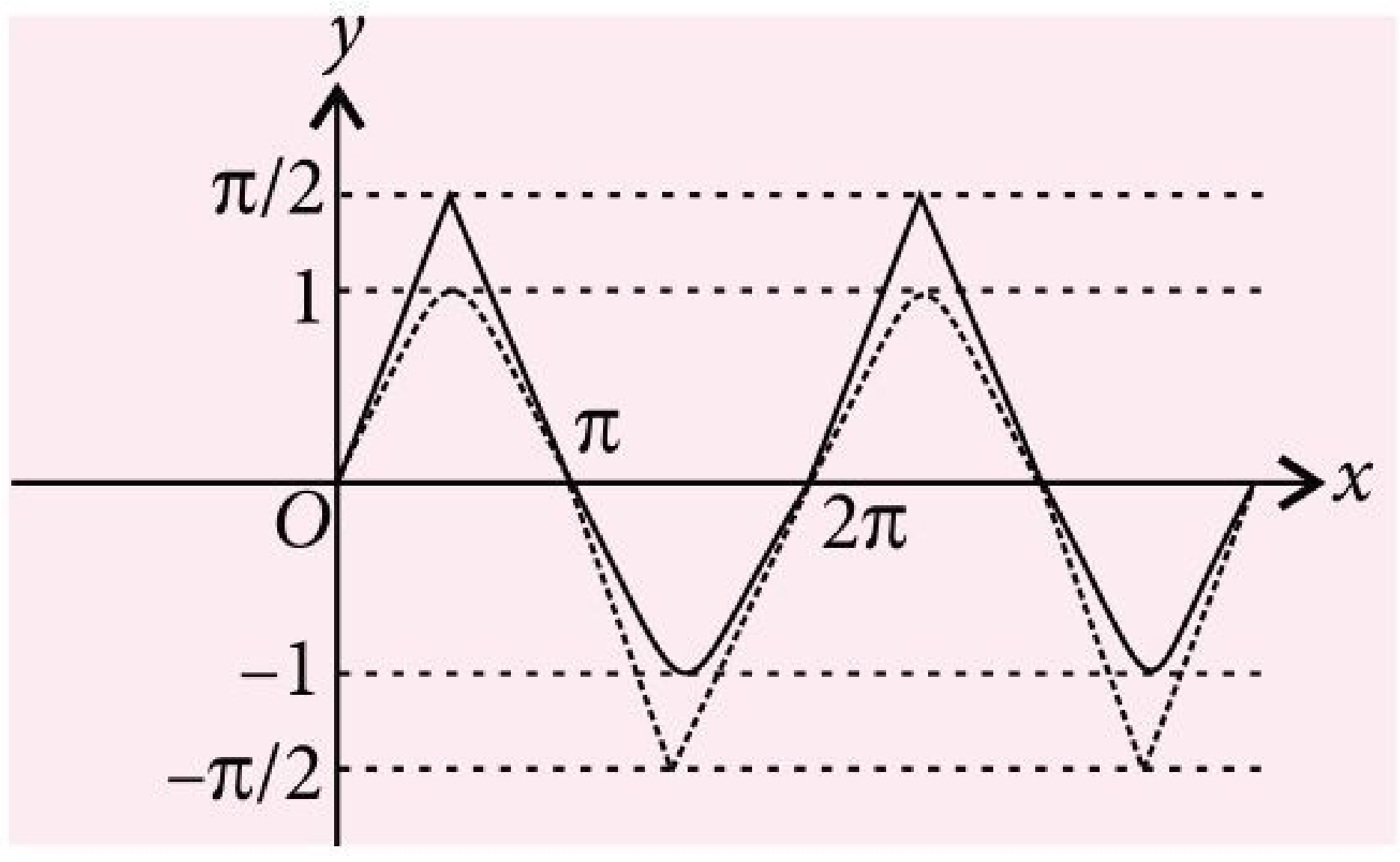
8. The integral can be divided as

$$= n\left[\int_0^{\frac{\pi}{2}} x\,dx + \int_{\frac{\pi}{2}}^{\pi} (\pi - x)\,dx + \int_{\pi}^{2\pi} \sin x\,dx\right]$$

$$= n\left[\frac{\pi^2}{8} + \frac{\pi^2}{2} - \frac{1}{2}\times\left(\pi^2 - \frac{\pi^2}{4}\right) - 2\right]$$

Solving, we get

$$I = n\times\frac{\pi^2 - 8}{4}$$



9. We take a point $P = (x_0, x_0^2)$ on the parabola. Then slope of tangent is $= 2x_0$

Hence slope of normal is $\frac{-1}{2x_0}$

So equation of normal is : $(x - x_0) = -2x_0(y - x_0^2)$

Solving it with the parabola, we get

$$x = x_0,\ x = \left(-\frac{1}{2x_0} - x_0\right)$$

So the other point is $Q = \left(-\frac{1}{2x_0} - x_0, \left(-\frac{1}{2x_0} - x_0\right)^2\right)$

Hence, the area bounded by PQ and the parabola is

$$\int_{\left(-\frac{1}{2x_0} - x_0\right)}^{x_0} \left(x_0^2 + \frac{1}{2} - \frac{x}{2x_0} - x^2\right)dx$$

(using the equation of normal)

Evaluating it, we get : $A = \frac{4}{3}\left(x_0 + \frac{1}{4x_0}\right)^3$

By applying AM-GM inequality, we get

$$\frac{x_0 + \frac{1}{4x_0}}{2} \ge \sqrt{\frac{1}{4}} \text{ or } \left(x_0 + \frac{1}{4x_0}\right)^3 \ge 1. \text{ Finally } A_{\min} = \frac{4}{3}$$

10. Let $L = \lim_{x\to 0}\left(\int_0^1 (by + a(1-y))^x\,dy\right)^{\frac{1}{x}}$

Let $I = \int_0^1 \left(by + a(1-y)\right)^x dy$

Let $by + a(1-y) = t$, $(b - a)\,dy = dt$

$$\Rightarrow I = \int \frac{t^x}{(b-a)}dt = \frac{t^{x+1}}{(x+1)(b-a)}$$

$$I = \left[\frac{(by + a(1-y))^{x+1}}{(x+1)(b-a)}\right] \text{ [computed at } y = 1 \text{ and } y = 0]$$

$$I = \frac{b^{x+1} - a^{x+1}}{(x+1)(b-a)} \Rightarrow L = \lim_{x\to 0}\left(\frac{b^{x+1} - a^{x+1}}{(x+1)(b-a)}\right)^{\frac{1}{x}}$$

$$ln(L) = \lim_{x\to 0}\frac{ln\left(\frac{\left(b^{x+1} - a^{x+1}\right)}{(x+1)(b-a)}\right)}{x}$$

As x approaches 0, denominator and numerator approaches 0.

Hence, we can use L-hospital's rule.

$$ln(L) = \lim_{x\to 0}\left(\frac{b^{x+1}ln(b) - a^{x+1}ln(a)}{(x+1)(b-a)} - \frac{1}{x+1}\right)$$

$$ln(L) = \frac{1}{b-a}ln\frac{b^b}{a^a} - 1 \Rightarrow L = \frac{1}{e}\left(\frac{b^b}{a^a}\right)^{\frac{1}{(b-a)}}$$

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SAMURAI SUDOKU

ANSWER · MARCH 2022

1	2	4	6	8	3	7	9	5				1	6	2	5	9	7	3	8	4
3	7	5	9	1	4	8	2	6				7	9	4	8	6	3	2	1	5
6	9	8	5	2	7	3	1	4				5	3	8	4	1	2	9	7	6
8	1	9	7	4	2	6	5	3				4	1	3	6	8	9	7	5	2
2	5	3	8	6	9	1	4	7				9	7	6	1	2	5	4	3	8
4	6	7	1	3	5	9	8	2				2	8	5	3	7	4	6	9	1
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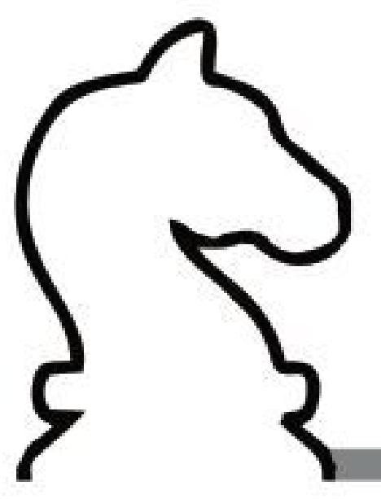
5	8	2	3	9	6	4	7	1	6	3	5	8	2	9	7	5	6	1	4	3
						8	2	5	9	6	3	7	1	4						
						1	9	3	4	8	7	2	6	5						
						6	4	7	1	5	2	9	8	3						
2	1	3	8	9	6	7	5	4	2	9	6	1	3	8	6	2	7	5	4	9
4	5	9	1	2	7	3	8	6	5	7	1	4	9	2	5	1	8	7	6	3
7	8	6	4	5	3	9	1	2	3	4	8	5	7	6	4	3	9	2	8	1
3	6	4	5	7	1	8	2	9				9	6	1	8	5	3	4	7	2
9	7	1	2	6	8	5	4	3				2	4	3	7	6	1	8	9	5
5	2	8	3	4	9	6	7	1				8	5	7	9	4	2	3	1	6
8	9	7	6	1	2	4	3	5				7	1	9	2	8	5	6	3	4
1	3	5	9	8	4	2	6	7				3	2	4	1	7	6	9	5	8
6	4	2	7	3	5	1	9	8				6	8	5	3	9	4	1	2	7

Challenging PROBLEMS ON JEE Main



SECTION-1 (SINGLE ANSWER TYPE)

1. Number of solutions satisfying the equation, $3x^2 - 2x^3 = \log_2(x^2+1) - \log_2 x$ is

(a) 1 (b) 2 (c) 3 (d) none

2. Let, $f(x) = x\sin\pi x - \dfrac{\cos\pi x}{\pi}$ $x > 0$. Then for all natural numbers n, $f'(x)$ vanishes at

(a) a unique point in the interval $\left(n, n+\dfrac{1}{2}\right)$

(b) a unique point in the interval $\left(n+\dfrac{1}{2}, n+1\right)$

(c) two points in the interval $\left(n,\ n+\dfrac{1}{2}\right)$

(d) two points in the interval $\left(n+\dfrac{1}{2}, n+1\right)$

3. In a triangle with one side 3 units, a circle of radius 1 is inscribed touching all sides then the minimum area of triangle equals

(a) $\dfrac{27}{5}$ (b) $\dfrac{24}{5}$ (c) 5 (d) $\dfrac{20}{5}$

4. If $x \in R$, the maximum value of

$$\sqrt{x^4 - 3x^2 - 6x + 13} - \sqrt{x^4 - x^2 + 1}$$ equals

(a) $\sqrt{7}$ (b) $\sqrt{10}$ (c) 3 (d) 4

5. If x, y, z are positive real numbers then the minimum value of $\dfrac{x^4 + y^4 + z^2}{xyz}$ is k_1 and minimum value of $\dfrac{\sec^4\alpha}{\tan^2\beta} + \dfrac{\sec^4\beta}{\tan^2\alpha}, \alpha, \beta \in \left(0, \dfrac{\pi}{2}\right)$ is k_2 then sum of digits of $k_1^2 + k_2^2$ equals

(a) 8 (b) 12 (c) 10 (d) 9

6. Let $f : R \to R^+$ be a differentiable function satisfying $f'(x) = 2f(x)\ \forall x \in R$. Also $f(0) = 1$ and $g(x) = f(x) \cdot \cos^2 x$. If n_1 represent number points of local maxima of $g(x)$ in $[-\pi, \pi]$ and n_2 is the number of points of local minima of $g(x)$ in $[-\pi, \pi]$ and n_3 is the number of points in $[-\pi, \pi]$ where $g(x)$ is attains its global minimum value ,then find the value of $(n_1 + n_2 + n_3)$

(a) 7 (b) 8 (c) 5 (d) 4

7. Let the function $f : [-4, 4] \to [-1, 1]$ be defined implicitly by the equation $x + 5y - y^5 = 0$, the area of triangle formed by tangent and normal to $f(x)$ at $x = 0$ and the line $y = 5$ is

(a) 70 (b) 65 (c) 75 (d) none

8. If the point on $y = x\tan\alpha - \dfrac{ax^2}{2u^2\cos^2\alpha}, 0 < \alpha < \dfrac{\pi}{2}$ where the tangent is parallel to $y = x$ has an ordinate $\dfrac{u^2}{4a}$, then the value of α is

(a) $\pi/2$ (b) $\pi/6$ (c) $\pi/3$ (d) $\pi/4$

9. If the sum of the maximum value and the minimum value of $y = \log_2(x^4 + x^2 + 1) - \log_2(x^4 + x^3 + 2x^2 + x + 1)$ can be expressed in form $((\log_2 m) - n)$, where m and n are relatively prime positive integers then the value of $(m + n)$ is

(a) 5 (b) 2 (c) 3 (d) 4

10. Consider the function $f(x) = \dfrac{x^2 - 21x + 1}{x^2 + 1}$, then for real x and y the maximum value of $|f(x) - f(y)|$ is

(a) 20 (b) 21 (c) 40 (d) 42

11. A function $f : R \to R$ such that for all $x \in R$, $f\left(x - \dfrac{b}{a}\right) + 2x \le \dfrac{a}{b}x^2 + \dfrac{2b}{a} \le f\left(x + \dfrac{b}{a}\right) - 2x$ where a, b be

By : Tapas Kr. Yogi, Visakhapatnam Mob : 09533632105

fixed positive real numbers then minimum value of $f(1)$ is

(a) 1 (b) 1/2 (c) 2 (d) 3/2

SECTION-2 (MULTIPLE ANSWER TYPE)

12. If the function

$$f(x)=(\sqrt{4-x^2}-3)^2+(\sqrt{4-x^2}+1)^3$$

then which of the following is/are correct? ([·] denotes GIF)

(a) $f(x)$ has greatest value 28

(b) $f(x)$ has least value $k \Rightarrow [k] = 9$

(c) The slope of tangent of $f(x)$ at $x=1$ equals $-(8+2\sqrt{3})$

(d) $f'(2) = 0$

13. Let $f: R \to R$ be defined as $f(x) = \sin x + ax + b (a, b, \in R)$. Then the equation $f(x) = 0$ has

(a) only one real root which is positive if $a > 1$, $b < 0$

(b) only one real root which is negative if $a > 1$, $b > 0$

(c) only one real root which is negative if $a < -1$, $b < 0$

(d) only one real root which is positive if $a < -1$, $b < 0$

14. Let f be a real valued function on R defined as, $f(x) = x^4(1 - x)^2$ then which of the following statement (s) is (are) correct?

(a) $f'(c) = 0$ for some $c \in (0, 1)$

(b) $f''(x)$ vanishes exactly twice in R.

(c) $f(x)$ is an even function

(d) Monotonic increasing in $\left(0,\frac{2}{3}\right) \cup (1,\infty)$

15. Let $f(x)$ be a twice differentiable function such that $f''(x) > 0$ in $[0, 2]$. Then

(a) $f(0)+f(2)=2f(c)$ for atleast one $c \in (0,2)$

(b) $f(0)+f(2)<2f(1)$

(c) $f(0) + f(2) > 2f(1)$

(d) $2f(0)+f(2)>3f\left(\frac{2}{3}\right)$

16. Consider $f(x)=\begin{cases} x^3-3x+3 \ ; & x\in[-2,0) \\ x+1 \ ; & x\in[0,5] \\ 1-e^{-x} \ ; & x>5 \end{cases}$ then

(a) $f(x)$ has local maxima at two points

(b) $f(x)$ has no local minima

(c) global minima of $f(x)$ does not exist

(d) global maxima value of $f(x)$ is 6

17. Which of the following is true?

(a) The least natural number 'a' for which $x + ax^{-2} > 2, \forall\ x \in (0, \infty)$ is 2

(b) The least value of the expression $x^2 + 4y^2 + 3z^2 - 2x - 12y - 6z + 14$ is 2.

(c) If $\sin\theta + \cos\theta = 1$ then the minimum value of $(1 + \operatorname{cosec}\theta)(1 + \sec\theta)$ is 9.

(d) If $a, b \in R$ are distinct numbers satisfying, $|a - 1| + |b - 1| = |a| + |b| = |a + 1| + |b + 1|$ then the minimum value of $|a - b|$ is 2.

18. If $|f(x)|=|ax^2+bx+c|\le 1$ for $|x| \le 1$ $(a, b, c \in R)$ and $\frac{8}{3}a^2+2b^2$ is maximum then

(a) $b = 0$ (b) $|a + c| = 1$

(c) $ac = 1$ (d) $\left|\frac{c}{a}\right|=\frac{1}{2}$

19. Let $f(x) = x^3 + ax^2 + bx + c$ where $a, b, c \in R$ then which of the following statement(s) is (are) correct?

(a) If the equation $f(x) = 0$ has exactly one real root then $f(x)$ must be strictly increasing on R.

(b) If $f(x)$ has a negative point of local minimum then both roots of equation $f'(x) = 0$ must be negative and distinct.

(c) If $f(x_1) \cdot f(x_2) < 0$, $x_1 < x_2$, then the equation $f(x) = 0$ has atleast one real root in (x_1, x_2).

(d) $f(x)$ possesses exactly one point of inflection.

20. Let $f(x)=\begin{cases} 0 & , \quad x=0 \\ (1-x)^{\alpha}x^{\beta}(1-\cos 2\pi x) & , \quad 0<x<1 \\ 0 & , \quad x=1 \end{cases}$.

If Rolle's theorem is applicable to $f(x)$ for $x \in [0, 1]$ then

(a) $\alpha \in R$ (b) $\beta \in R$

(c) $\beta \in (-2, \infty)$ (d) $\alpha \in (-2, \infty)$

SOLUTIONS

1. (a) : Note that L.H.S has a maximum value 1 at $x = 1$ but R.H.S side has a minimum value 1 at $x = 1$ $\Rightarrow x = 1$ can be the only solution

2. (b) : $f'(x) = 0 \Rightarrow \tan\pi x = -\pi x/2$

$$\Rightarrow \left(n+\frac{1}{2}\right)\pi < \pi x < (n+1)\pi$$

$$\Rightarrow n+\frac{1}{2}<x<n+1$$

Unique solution is $\left(n+\frac{1}{2}, n+1\right)$

3. (a) :

$$\left.\begin{aligned}BC&=a\\CA&=b\\AB&=c\end{aligned}\right\}\left.\begin{aligned}AD&=AF=x\\FC&=CE=y\\BD&=BE=z\end{aligned}\right\}\therefore\left.\begin{aligned}a&=y+z\\b&=x+y\\c&=x+z\end{aligned}\right\}\begin{aligned}&\therefore y+z=3\\&r=\frac{\Delta}{s}\Rightarrow\Delta=s\end{aligned}$$

$$\Delta=\sqrt{(x+3)xyz}\Rightarrow x+3=\sqrt{(x+3)xyz}\Rightarrow x=\frac{3}{yz-1}$$

$$\therefore\ \Delta=\sqrt{\left(\frac{3}{(yz-1)}+3\right)\left(\frac{3}{yz-1}\right)yz}=\frac{3yz}{yz-1}=3+\frac{3}{yz-1}$$

$$\geq 3+\frac{3}{\left(\frac{y+z}{2}\right)^2-1}=\frac{3}{\left(\frac{3}{2}\right)^2-1}+3=\frac{27}{5}$$

4. (b) : We have,

$$\sqrt{(x^2-2)^2+(x-3)^2}-\sqrt{(x^2-1)^2+(x-0)^2}$$

$P(x, x^2)$, $A(3, 2)$, $B(0, 1)$

$$\Rightarrow\ |PA|-|PB|\leq|AB|=\sqrt{10}$$

5. (d) : Use *AM-GM* for $x^4, y^4, \frac{z^2}{2}, \frac{z^2}{2}, K_1=2\sqrt{2}$

Also use $a^2+b^2\geq 2ab$

$$\Rightarrow\ GE\geq\frac{2\sec^2\alpha\sec^2\beta}{\tan\beta\times\tan\alpha}\geq\frac{8}{\sin 2\alpha\sin 2\beta}\geq 8$$

$k_1^2+k_2^2=8^2+8=72\ \Rightarrow$ sum of digits = 9

6. (b) : Given $f'(x)=2f(x)$

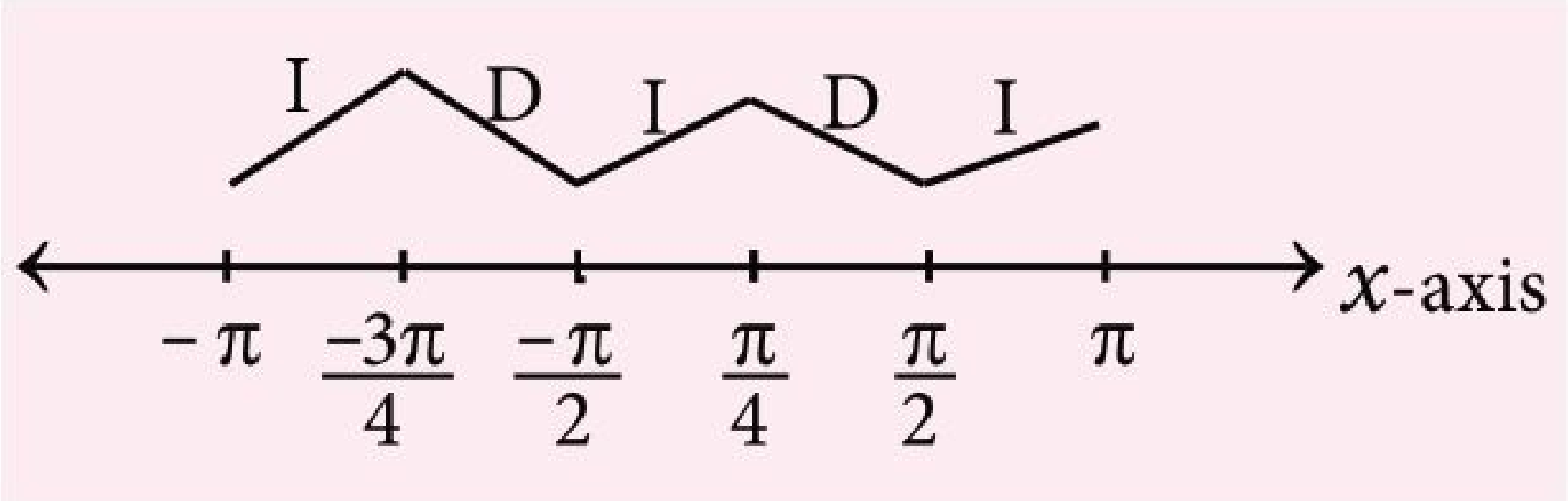
$$\therefore\ \frac{f'}{f}=2\ \Rightarrow\ f(x)=Ae^{2x}$$

$f(0)=1=A$

$\therefore\ f(x)=e^{2x}$

Now, $g(x)=e^{2x}\cdot\cos^2x$

$g'(x)=e^{2x}(-2\cos x\sin x+2\cos^2x)$



$g'(x)=2\cos x e^{2x}(\cos x-\sin x)$

$$g'(x)=0\Rightarrow x=\frac{-3\pi}{4},\frac{\pi}{4},\frac{\pi}{2},\frac{-\pi}{2}$$

$\therefore$ points of maxima are $\frac{-3\pi}{4},\frac{\pi}{4}$ and π points of minima are $-\pi,\frac{-\pi}{2},\frac{\pi}{2}$

And global minimum value occurs at $\frac{\pm\pi}{2}$ which is zero

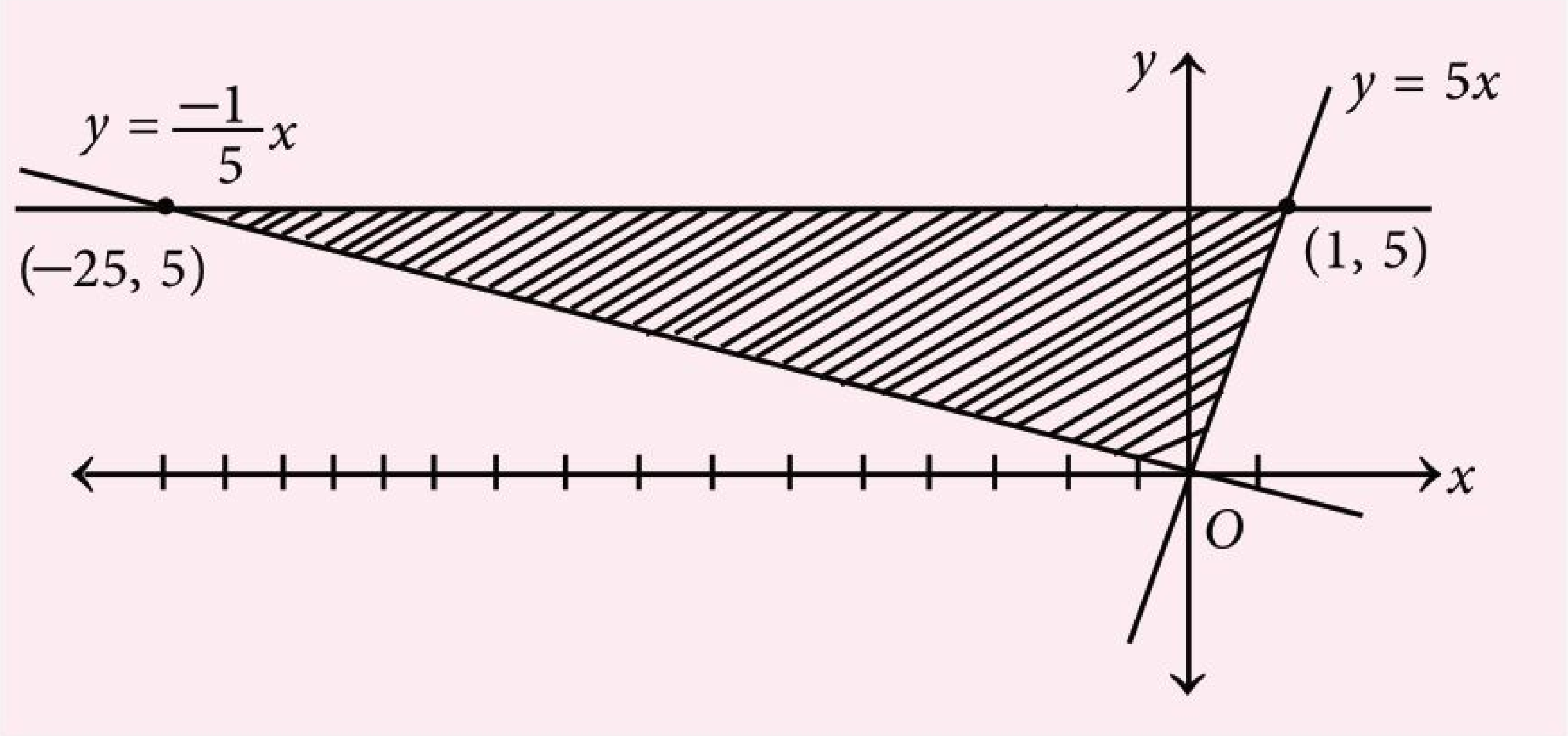
Hence, $n_1=3, n_2=3, n_3=2\Rightarrow n_1+n_2+n_3=8$

7. (b) : Given $x+5y-y^5=0\ \Rightarrow\ \frac{dy}{dx}=\left(\frac{1}{5y^4-5}\right)$

Now, $x=0$, $y=\pm(5)^{\frac{1}{4}}$ or $y=0$

But, $y=\pm(5)^{\frac{1}{4}}$ (Rejected)

So, slope of tangent at $(0, 0)=\frac{-1}{5}$



$\Rightarrow$ Equation of tangent is $y=\frac{-x}{5}$ ...(1)

Now, equation of normal is $y=5x$ ...(2)

Also given line is $y=5$ ...(3)

Clearly, area of triangle $=\frac{1}{2}(26)(5)=13(5)=65$

8. (c) : Given $m=1\ \Rightarrow\ \tan\alpha-\frac{ax}{u^2\cos^2\alpha}=1$

$$\Rightarrow\ x=\frac{(\tan\alpha-1)}{a}u^2\cos^2\alpha$$

Substitute x and y values in given equation

$$\frac{u^2}{4a}=\frac{u^2}{a}\left[\sin^2\alpha-\frac{1}{2}\right]\ \Rightarrow\ \alpha=\frac{\pi}{3}$$

9. (a) : $y=\log_2\left(\frac{(x^2+x+1)(x^2-x+1)}{x^4+x^3+2x^2+x+1}\right)$

$$=\log_2\left(\frac{x^2-x+1}{x^2+1}\right)$$

Assume $z=\frac{x^2-x+1}{x^2+1}$

$\Rightarrow\ (z-1)x^2+x+(z-1)=0$, As $x\in R$, so

$$D\geq 0\Rightarrow 1-4(z-1)^2\geq 0\Rightarrow(z-1)^2\leq\frac{1}{4}\Rightarrow\frac{1}{2}\leq z\leq\frac{3}{2}$$

So, $y_{\min} = \log_2\left(\frac{1}{2}\right) = -1$

And $y_{\max} = \log_2\left(\frac{3}{2}\right) = (\log_2 3 - 1)$

Hence $y_{\min} + y_{\max} = \log_2 3 - 2 \equiv ((\log_2 m) - n)$

∴ $m = 3, n = 2$

Hence $(m + n) = 3 + 2 = 5$

10. (b) : $f'(x) = \frac{21(x^2-1)}{(x^2+1)^2}$

So, $f(x)$ increases for $x \in (-\infty, -1] \cup [1, \infty)$ and decreases for $(-1, 1)$. So, $f_{\max} = f(-1)$ and $f_{\min} = f(1)$

So, for any x, y

$|f(x) - f(y)| \le |f(-1) - f(1)| \le 21$

11. (c) : Putting $y = x - \frac{b}{a}$ in the L.H.S. of the inequality, we have $f(y) \le \frac{a}{b}y^2 + \frac{b}{a}$ and while putting $y = x + \frac{b}{a}$ in the R.H.S. of the inequality given, we have

$f(y) \ge \frac{a}{b}y^2 + \frac{b}{a}$

Hence, $f(x) = \frac{a}{b}x^2 + \frac{b}{a}$.

12. (a, b, c) : $x \in [-2, 2] \Rightarrow a \in [0, 2]$ where $a = \sqrt{4 - x^2}$

$f(a) = (a - 3)^2 + (a + 1)^3$

$f'(a) = 3a^2 + 8a - 3 = (3a - 1)(a + 3)$

$\therefore\ f_{\min} = f(1/3) = \frac{64}{9} + \frac{64}{27} = \frac{256}{27}$

$f_{\max} = f(2) = 28$

$\frac{dy}{dx} = (6 + 8\sqrt{3})\frac{1}{2\sqrt{4-1}}(-2\times 1) \quad (x = 1 \Rightarrow a = \sqrt{3})$

$= -\left(\frac{6 + 8\sqrt{3}}{\sqrt{3}}\right) = -(8 + 2\sqrt{3})$

13. (a, b, c) : $f : R \rightarrow R$ and

$f(x) = \sin x + ax + b$

$\Rightarrow f'(x) = \cos x + a$

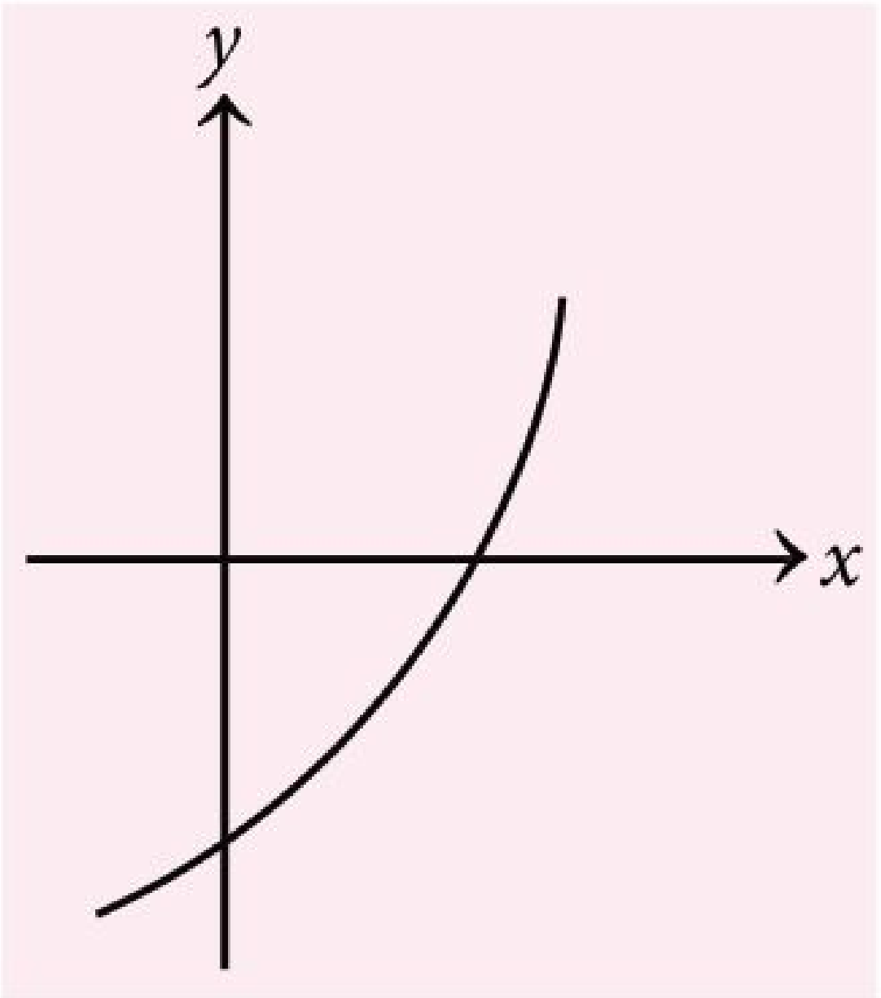
If $a > 1$, then f is increasing

If $a < -1$ then f is decreasing

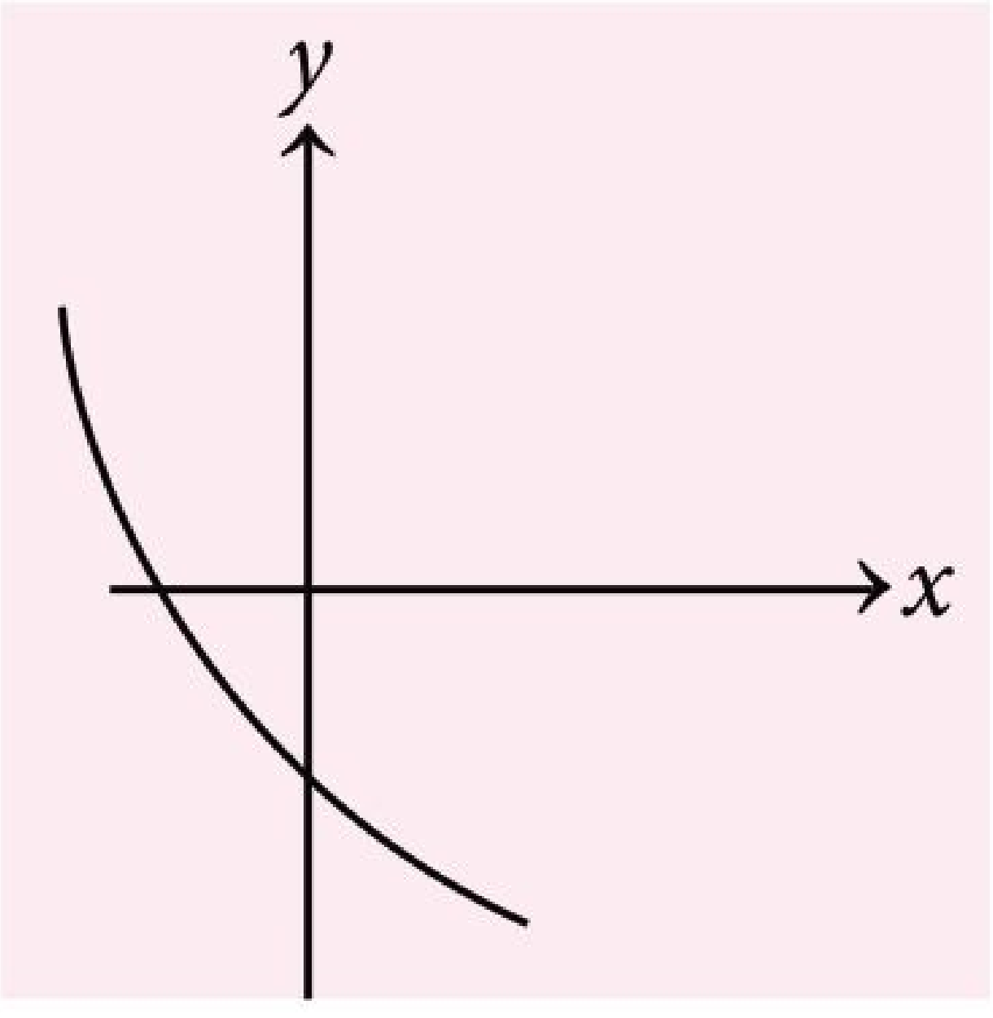
If $a > b$, $f(0) = b$ and

$b < 0 \Rightarrow f(0) < 0$

As shown in the diagram



∴ $f(x) = 0$ has only the +ve root hence (a) is correct



If $b > 0$ & $a < 1$, $f(0) > 0$ again f has only one negative root

If $a < -1$, $b < 0$ then f is decreasing

and $f(0) < 0 \Rightarrow f$ has one negative root.

14. (a d) : $f(x) = x(1 - x)^2$

$\Rightarrow f'(x) = 2x^3(x - 1)(3x - 2)$

Now, $f'(x) = 0 \Rightarrow x = 0, 2/3, 1$

$f''(x) = 2x^2(15x^2 - 20x + 6)$

$f''(0) = 0$ and $15x^2 - 20x + 6 = 0$ has two roots

16. (a, c, d) : $x \in [0, 2]$ and $f''(x) > 0$

$\Rightarrow$ $f'(x)$ is increasing in $[0, 2]$

$\Rightarrow$ LMVT is applicable and

Let $c_1 \in (0, 1)$ and $c_2 \in (1, 2)$

By LMVT, $f'(c_1) = \frac{f(1) - f(0)}{1 - 0}$; $f'(c_2) = \frac{f(2) - f(1)}{2 - 1}$

$f'(x)$ is increasing and $c_1 < c_2$

$\Rightarrow f'(c_1) < f'(c_2)$

$\Rightarrow f(1) - f(0) < f(2) - f(1) \Rightarrow f(0) + f(2) > 2f(1)$

Let $c_1 \in (0, 2/3)$ and $c_2 \in (2/3, 2)$

$f'(c_1) < f'(c_2)$

$\Rightarrow \frac{f(2/3) - f(0)}{\frac{2}{3} - 0} < \frac{f(2) - f(2/3)}{2 - \frac{2}{3}}$

$\Rightarrow 2f(0) + f(2) > 3f(2/3)$

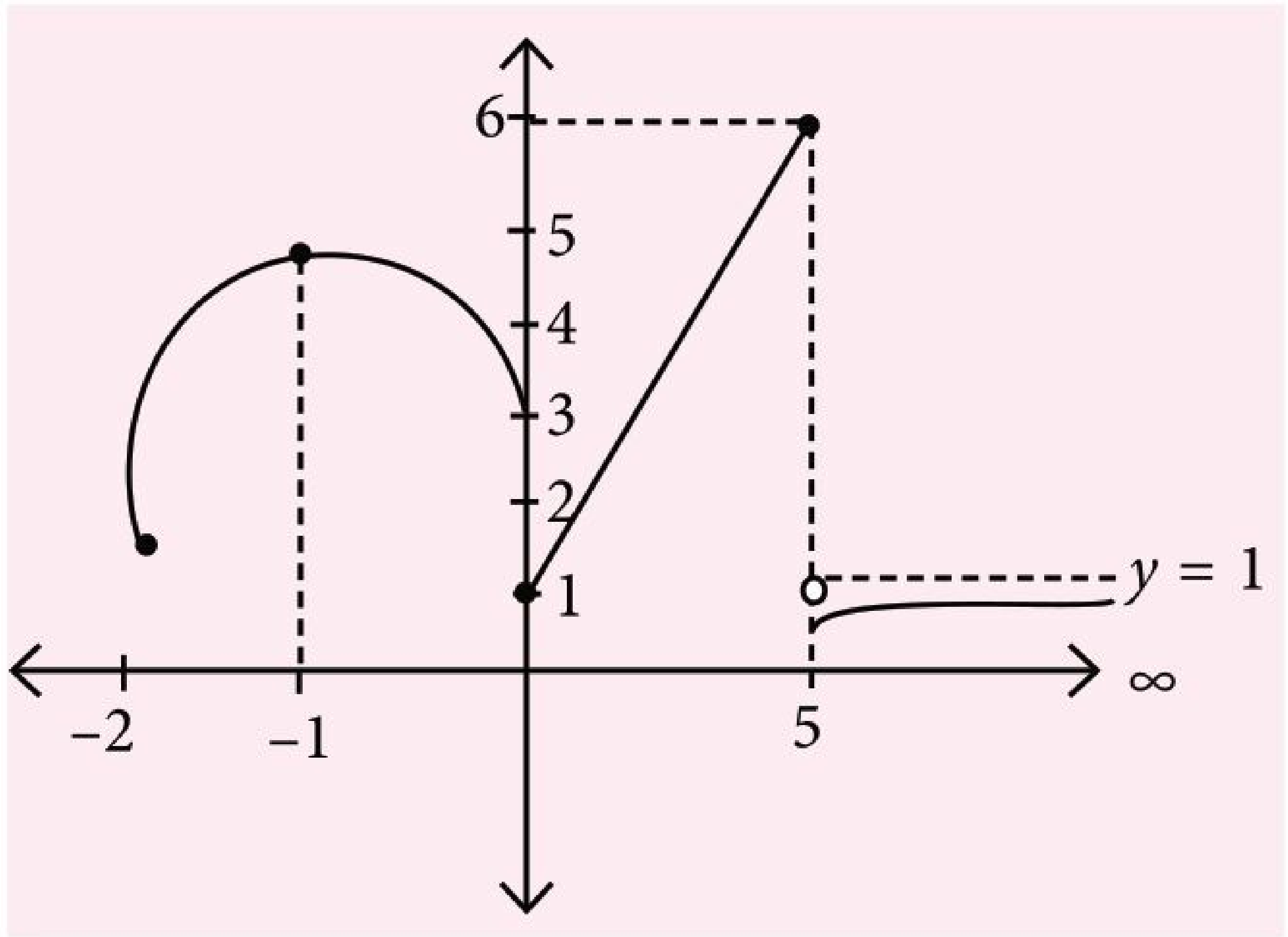
16. (a, c, d) : $f(x)$ is continuous in $[-2, 0)$

$\Rightarrow$ $f(x)$ has local maximum, at $x = -1$, and discontinuous at $x = 5$ and $x = 0$.

But $f(x)$ has local maximum at $x = 5$ $\left(\because\ f(5) > f(5^+)\right)$

$\Rightarrow$ global minima of $f(x)$ does not exist

$\Rightarrow$ global maxima value of $f(x) = 6$



17. (a c d) : (a) $x+\frac{a}{x^2}>2,\forall x\in(0,\infty)$

$$\Rightarrow \frac{x}{2}+\frac{x}{2}+\frac{a}{x^2}>2$$

∴ $AM \ge GM$

$$\Rightarrow \frac{\frac{x}{2}+\frac{x}{2}+\frac{a}{x^2}}{3}\ge\left(\frac{x}{2}\cdot\frac{x}{2}\cdot\frac{a}{x^2}\right)^{1/3}$$

$$\Rightarrow x+ax^{-2}\ge 3\frac{a^{1/3}}{4^{1/3}}>2 \quad \Rightarrow \frac{3^3a}{4}>8\Rightarrow a>\frac{32}{27}$$

(b) $f(x, y, z) = x^2 - 2x + 1 + 4y^2 - 12y + 9 + 3z^2 - 6z + 3 + 1$

$= (x - 1)^2 + (2y - 3)^2 + 3(z - 1)^2 + 1$

Hence least value of $f(x, y, z) = 1$, when $x=1, y=\frac{3}{2}, z=1$

(c) ∴ $AM \ge GM$

$$\Rightarrow \frac{\sin\theta+\cos\theta}{2}\ge\sqrt{\sin\theta\cos\theta}$$

$$\Rightarrow \sin\theta\cos\theta\le\frac{1}{4} \Rightarrow xy\le\frac{1}{4}$$

Let $\sin\theta = x$, $\cos\theta = y$ and $(1 + \operatorname{cosec}\theta)(1 + \sec\theta) \ge p$

$$\Rightarrow \left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)\ge p$$

$\Rightarrow (x + 1)(1 + y) \ge pxy \quad \Rightarrow 1 + x + y + xy \ge pxy$

$\Rightarrow 2 \ge (p - 1)xy$

$$\Rightarrow xy\le\frac{2}{p-1} \qquad \left(\text{But } xy\le\frac{1}{4}\right)$$

$$\Rightarrow \frac{2}{p-1}\le\frac{1}{4} \Rightarrow p-1\le 8\Rightarrow p\le 9$$

(d) Let $a < b$ and $f(x) = |x - a| + |x - b|, \forall x \in R$

So, $f(x)$ is decreasing in $(-\infty, a]$, constant in $[a, b]$ and increasing in $[b, \infty)$

We have, $f(0) = f(1) = f(-1)$

$\Rightarrow \{-1, 0, 1\} \in [a, b]$ ∴ $|a - b|_{\min} = 2$

18. (a, b, d) : $\frac{8}{3}a^2+2b^2=\frac{2}{3}[4a^2+3b^2]$

$$=\frac{2}{3}[4(a^2+b^2)-b^2]$$

Now, $|f(1)|\le 1, \quad |f(0)|\le 1, \quad |f(-1)|\le 1\Rightarrow |f(1)-f(0)|\le 2$

$\Rightarrow |a + b| \le 2$

Now, $|f(-1) - f(0)| \le 2 \quad \Rightarrow \quad |a - b| \le 2$

∴ $4(a^2 + b^2) - b^2 = 2((a + b)^2 + (a - b)^2) - b^2$

$\le 2 \times 8 - b^2$

∵ $|a + b| = 2, |a - b| = 2$

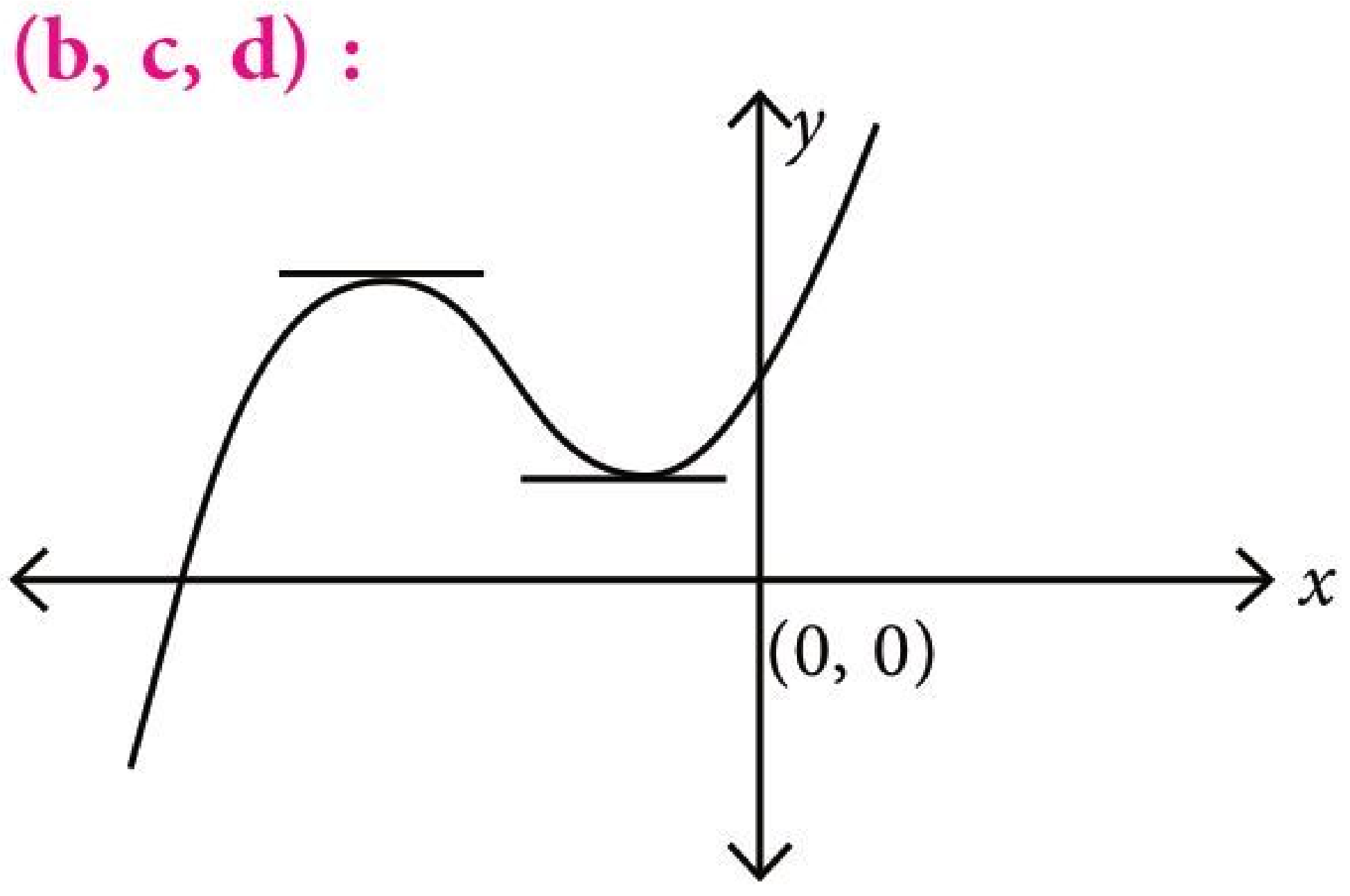
When $f(1) = 1, f(0) = -1, f(-1) = 1$

or $f(1)= -1, f(0) = 1, f(-1) = -1$

we get $b = 0$, $a = \pm 2$, $c = 1$

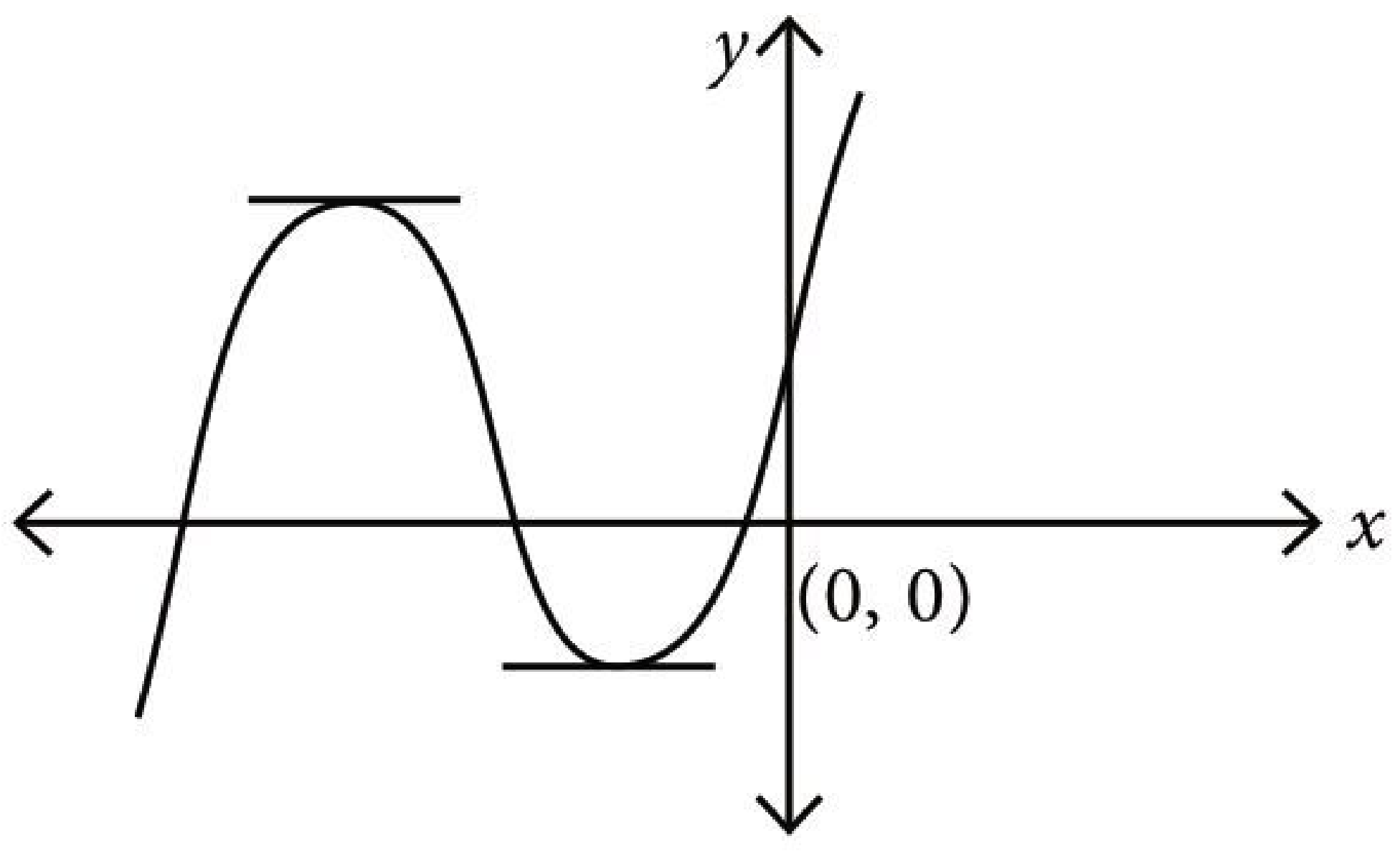
19. (b, c, d) :

(a)



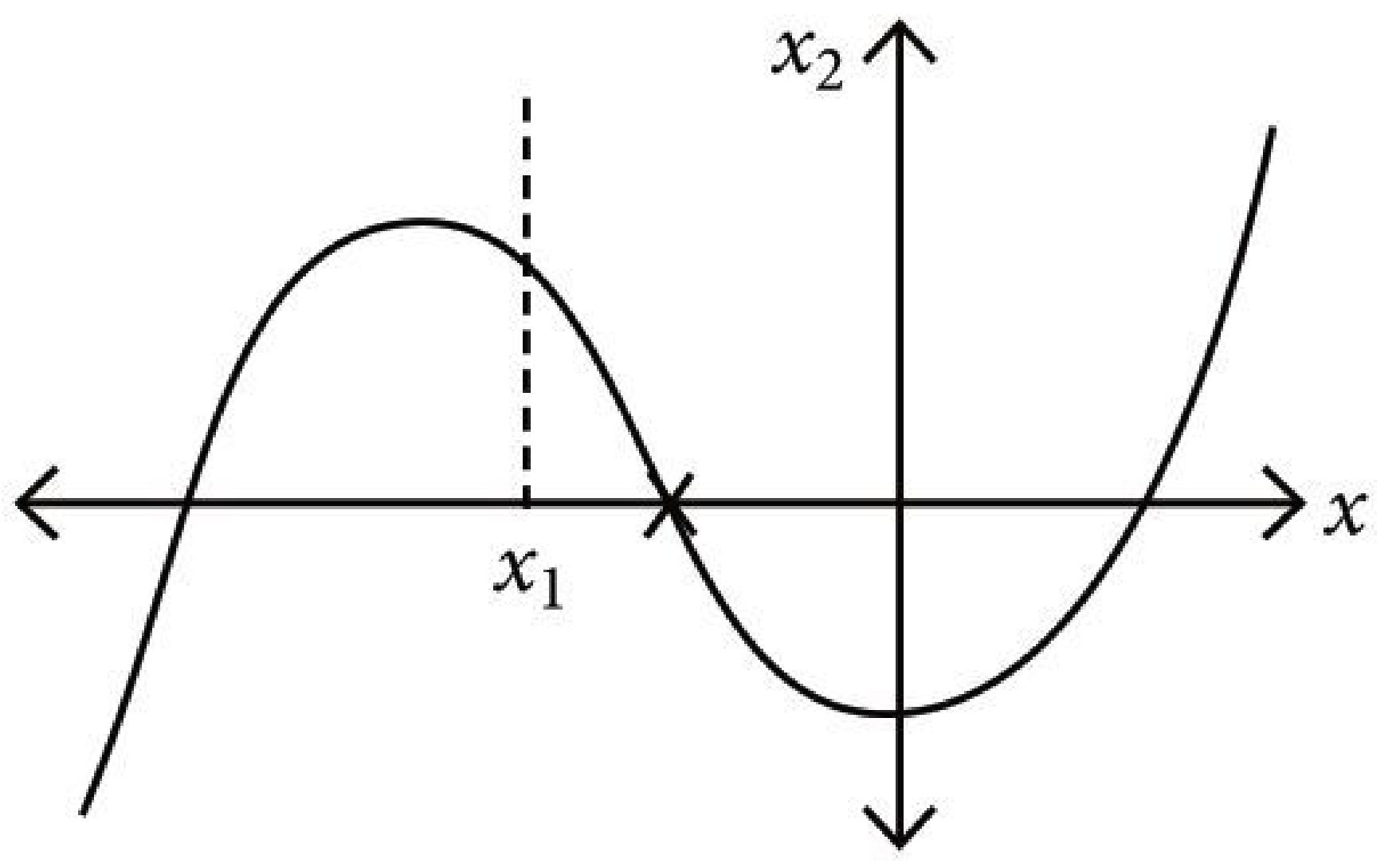
⇒ False

(b)



⇒ True

(c)



⇒ True

(d) $f''(x)=2(3x+a)=0\Rightarrow x=-\frac{a}{3}$ ⇒ true

and $f''(x)$ changes sign also.

20. (c, d) : Since $f(0) = 0$, $f(1) = 0$

$f(x)$ is continuous at $x = 0 \Rightarrow f(0^+) = f(0)$

$$\Rightarrow 0=\lim_{x\to 0^+}(1-x)^\alpha x^\beta(2\sin^2\pi x)$$

$$=\lim_{x\to 0^+}2\pi^2(1-x)^\alpha x^2\cdot x^\beta=2\lim_{x\to 0^+}\pi^2(1-x)^\alpha x^{\beta+2}$$

when $\beta + 2 > 0 \Rightarrow \beta > -2$

But for continuity at $x = 1$

$$f(1)=f(1^-)\Rightarrow 0=\lim_{h\to 0^+}h^\alpha(1-h)^\beta(1-\cos(2\pi-2\pi h))$$

$$\Rightarrow 0=\lim_{h\to 0^+}h^\alpha(1-h)^\beta(2\sin^2 h\pi)$$

$$\Rightarrow \lim_{h\to 0^+}h^\alpha(1-h)^\beta\frac{2(\sin^2\pi h)}{(\pi h)^2}(\pi h)^2=0$$

$$\Rightarrow 2\pi^2\lim_{h\to 0^+}h^{\alpha+2}(1-h)^\beta=0 \Rightarrow \alpha+2>0 \Rightarrow \alpha>-2$$

(as $f(x)$ is derivable in $(0, 1)$

Therefore Rolle's theorem is applicable if $\alpha > -2$, $\beta > -2$

⁂

OLYMPIAD CORNER

1. Consider the infinite sum

$$S=\frac{a_0}{10^0}+\frac{a_1}{10^2}+\frac{a_2}{10^4}+\frac{a_3}{10^6}+\ldots,$$

where the sequence $\{a_n\}$ is defined by $a_0 = a_1 = 1$, and the recurrence relation $a_n = 20a_{n-1} + 12a_{n-2}$ for all positive integers $n \geq 2$. If $\sqrt{S}$ can be expressed in the form $\frac{a}{\sqrt{b}}$, where a and b are relatively prime positive integers, determine the ordered pair (a, b).

2. M is an interior point of a triangle ABC. Bisectors of interior angles BMC, CMA, AMB intersect BC, CA, AB respectively at X, Y, Z. Show that AX, BY, CZ are concurrent; if P is the point of concurrence and

$$\frac{PA}{PX}\cdot\frac{PB}{PY}\cdot\frac{PC}{PZ}=8$$

show that M is the circumcenter of ΔABC.

3. Let B_1 and C_1 be points on the sides AC and AB of triangle ABC. Lines BB_1 and CC_1 intersect at point D. Prove that a circle can be inscribed inside quadrilateral AB_1DC_1 if and only if the incircles of the triangles ABD and ACD are tangent to each other.

4. Ten players participate in a ping-pong tournament, in which every two players play against each other exactly once. If player i beats player j, player j beats player k, and player k beats player i, then the set $\{i, j, k\}$ is called a triangle. Let w_i and l_i denote the number of games won and lost, respectively, by the i^{th} player. Suppose that, whenever i beats j, then $l_i + w_j \geq 8$. Prove that there are exactly 40 triangles in the tournament.

5. Let p be a prime number such that $p \equiv 1 \pmod 4$.

Evaluate, $$\sum_{k=1}^{p-1}\left(\left\lfloor\frac{2k^2}{p}\right\rfloor-2\left\lfloor\frac{k^2}{p}\right\rfloor\right).$$

6. A hexagon is inscribed in a circle with radius r. Two of its sides have length 1, two have length 2 and the last two have length 3. Prove that r is a root of the equation $2r^3 - 7r - 3 = 0$.

7. Let n be a fixed positive integer. Show that for any nonnegative integer k, the equation

$$x_1^3+x_2^3+\ldots+x_n^3=y^{3k+2}$$

has infinitely many solutions in positive integers x_i and y.

8. Find the number of all natural numbers n, $4 \leq n \leq 1023$, whose binary representations do not contain three consecutive equal digits.

SOLUTIONS

1. We have

$$S-\frac{20S}{10^2}-\frac{12S}{10^4}=\left(\frac{a_0}{10^0}+\frac{a_1}{10^2}+\frac{a_2}{10^4}+\frac{a_3}{10^6}+\ldots.\right)$$

$$-\left(\frac{20a_0}{10^2}+\frac{20a_1}{10^4}+\frac{20a_2}{10^6}+\frac{20a_3}{10^8}+\ldots.\right)$$

$$-\left(\frac{12a_0}{10^4}+\frac{12a_1}{10^6}+\frac{12a_2}{10^8}+\frac{12a_3}{10^{10}}+\ldots.\right)$$

$$=\frac{a_0}{10^0}+\frac{a_1}{10^2}-\frac{20a_0}{10^2}+\frac{a_2-20a_1-12a_0}{10^4}$$

$$+\frac{a_3-20a_2-12a_1}{10^6}+\frac{a_4-20a_3-12a_2}{10^8}+\ldots.$$

Since, $a_n - 20a_{n-1} - 12a_{n-2} = 0$ for all positive integers $n \geq 2$, we have

$$S-\frac{20S}{10^2}-\frac{12S}{10^4}=\frac{a_0}{10^0}+\frac{a_1}{10^2}-\frac{20a_0}{10^2},$$

and substituting in $a_0 = a_1 = 1$, we have

$$\frac{7988S}{10000}=\frac{81}{100},\ \text{so}\ S=\frac{2025}{1997}.$$

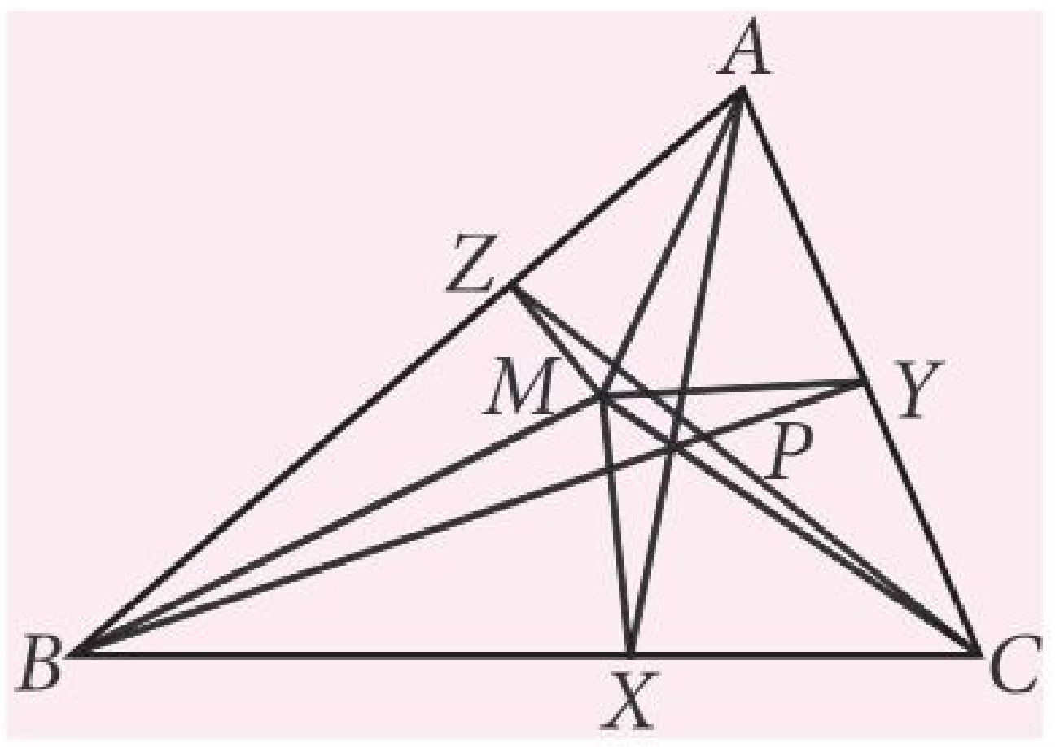
Hence, $\sqrt{S}=\frac{45}{\sqrt{1997}}$, and so the desired ordered pair is $(a, b) = (45, 1997)$.

2. Since MX is the internal bisector of $\angle BMC$, $\frac{BX}{XC}=\frac{MB}{MC}$; similarly, $\frac{CY}{YA}=\frac{MC}{MA}$ and $\frac{AZ}{ZB}=\frac{MA}{MB}$.

Therefore, $\frac{BX}{XC}.\frac{CY}{YA}.\frac{AZ}{ZB}=1$.

Hence by Ceva's theorem, AX, BY, CZ are concurrent.

Let $\Delta BPC = t_1$, $\Delta APC = t_2$ and $\Delta APB = t_3$.



Then, $\frac{PA}{PX}=\frac{\Delta APB}{\Delta BPX}=\frac{\Delta APC}{\Delta PXC}=\frac{\Delta APB+\Delta APC}{\Delta BPX+\Delta PXC}=\frac{t_2+t_3}{t_1}$.

Similarly, $\frac{PB}{PY}=\frac{t_3+t_1}{t_2}$ and $\frac{PC}{PZ}=\frac{t_1+t_2}{t_3}$.

Now,

$$8=\frac{(t_2+t_3)(t_3+t_1)(t_1+t_2)}{t_1t_2t_3}\ge\frac{2\sqrt{t_2t_3}2.\sqrt{t_3t_1}.2\sqrt{t_1t_2}}{t_1t_2t_3}$$

(Equality holds only when $t_1 = t_2 = t_3 = 8$)

Thus, $t_1=t_2=t_3$. Now, $\frac{BX}{XC}=\frac{\Delta ABX}{\Delta ACX}$

$$=\frac{\Delta PBX}{\Delta PCX}=\frac{\Delta ABX-\Delta PBX}{\Delta ACX-\Delta PCX}=\frac{t_3}{t_2}=1$$

i.e., $BX = XC$. Since MB is the bisector of $\angle BMC$, it follows that $MB = MC$. Similarly $MC = MA$. Therefore, M is the circumcenter. (Note that P is the centroid).

3. Let the incircle of triangle ABD be tangent to $\overline{AD}$ at T_1 and let the incircle of triangle ACD be tangent to $\overline{AD}$ at T_2. Then

$$DT_1=\frac{1}{2}(DA+DB-AB)$$

and $DT_2=\frac{1}{2}(DA+DC-AC)$

First suppose a circle can be inscribed inside AB_1DC_1. Let it be tangent to sides AB_1, B_1D, DC_1, C_1A at points E, F, G, H, respectively. Using equal tangents, we have

$AB - BD = (AH + HB) - (BF - DF)$

$= (AH + BF) - (BF - DF) = AH + DF$

and similarly $AC - CD = AE + DG$. Also, $AH + DF = AE + DG$ by equal tangents, implying that $AB - BD = AC - CD$ and thus $DA + DB - AB = DA + DC - AC$. Therefore $DT_1 = DT_2$, $T_1 = T_2$, and the two given incircles are tangent to each other. Next suppose the two incircles are tangent to each other. Then $DA + DB - AB = DA + DC - AC$. Let ω be the incircle of ABB_1, and let D' be the point on $\overline{BB_1}$ (different from B_1) such that line CD' is tangent to ω. Suppose, by way of contradiction, that $D \neq D'$. From the result in the last paragraph, we know that the incircles of triangles ABD' and ACD' are tangent and hence $D'A + D'B - AB = D'A + D'C - AC$. Because $DB - AB = DC - AC$ and $D'B - AB = D'C - AC$, we must have $DB - D'B = DC = D'C$ by subtraction. Thus $DD' = |DB - D'B| = |DC - D'C|$. The triangle inequality then fails in triangle $DD'C$, a contradiction. This completes the proof.

4. Let P_i denote the i^{th} player $i = 1, 2,\ldots, 10$. Without loss of generality, assume that $w_1 \le w_2 \le \ldots \le w_{10}$. Let $M = \max_{1\le i\le 10} w_i = w_{10}$ and $m = \min_{1\le i\le 10} w_i = w_1$. Since $w_i + l_i = 9$ for all i, the condition $l_i + w_j \ge 8$, whenever P_i beats P_j, is equivalent to

$$w_i \le w_j + 1. \quad \text{...(i)}$$

Clearly $\sum_{i=1}^{10} w_i=\binom{10}{2}=45$. ...(ii)

We first show that

$(w_1, w_2, \ldots, w_{10}) = (4, 4, 4, 4, 4, 5, 5, 5, 5, 5)$

Since P_1 beats m players and loses to $9 - m$ others, we get from (i) and (ii) that

$$45 \le m + Mm + (9 - m)(m + 1)$$

or $36 \le (M - m + 9)m$. ...(iii)

Similarly, since P_{10} beats M players and loses to $9 - M$ others, we have $45 \ge M + M(M - 1) + (9 - M)m = M^2 + (9 - M)m$. ...(iv)

Since $M \le 9$, from (iii) we obtain $36 \le (18 - m)m$, which implies $m \ge 3$. From (iii) we get $M^2 \le 45$, *i.e.*, $M \le 6$. If $M = 6$, then from (iv), $3m \le 9$ or $m \le 3$ and so $m = 3$. Similarly, if $m = 3$ then from (iii) $M \ge 6$ and so $M = 6$. Since 10×45, $M \neq m$. Therefore, the only possibilities are $(M, m) = (6, 3)$ and $(5, 4)$.

Note : At this point, Lau listed all 10-tuples $(w_1, w_2, \ldots, w_{10})$ satisfying $(M, m) = (6, 3)$ and (2), and then showed that none were possible. But the case $(M, m) = (6, 3)$ can be eliminated as follows. If $m = w_1 = 3$, then six players beat P_1, so by (i) at least seven of the w_i's (including w_1) are ≤ 4. If $M = w_{10} = 6$, then P_{10} beats six players, so by (i) atleast seven of the w_i's (including w_{10}) are ≥ 5.

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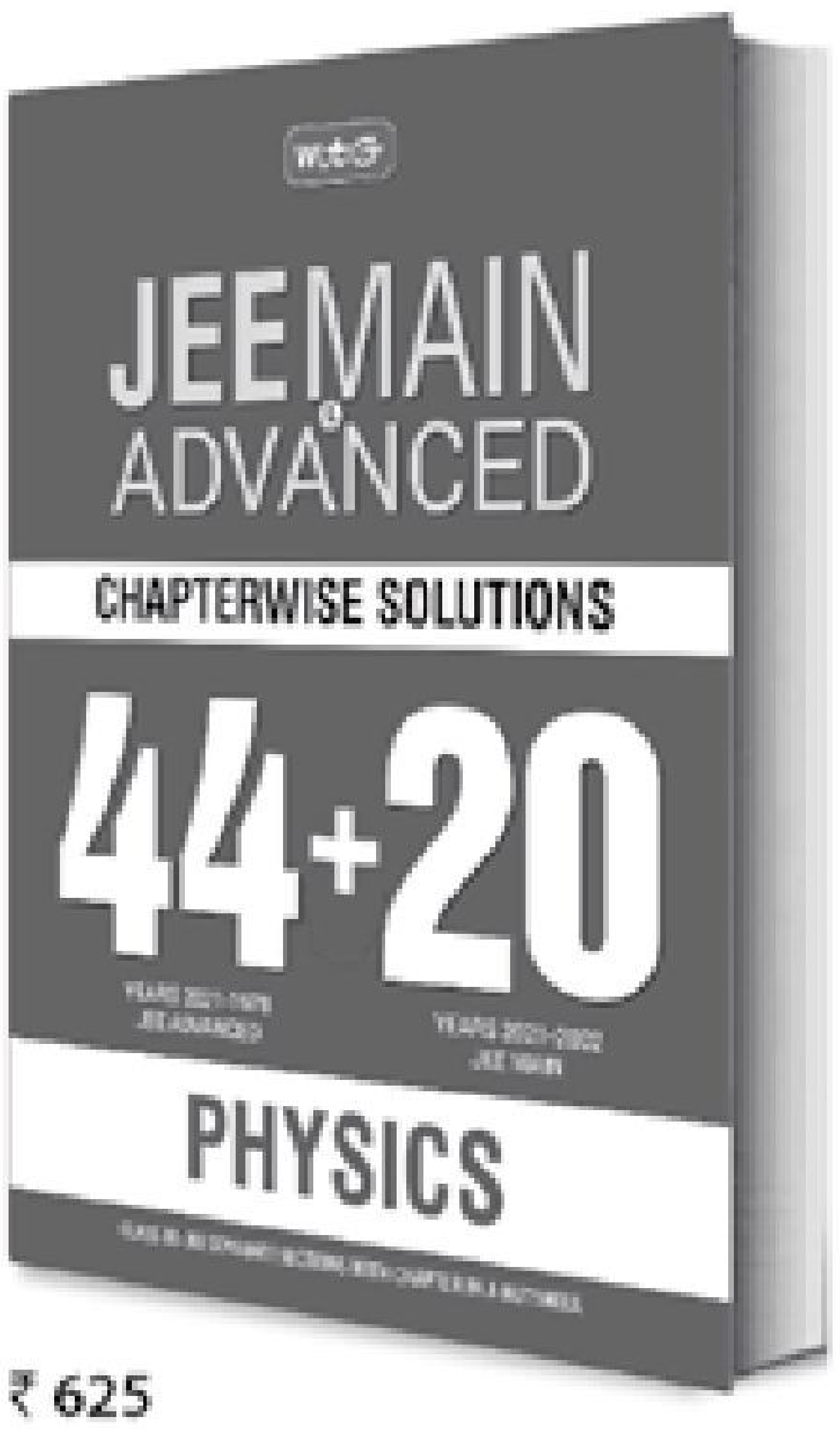
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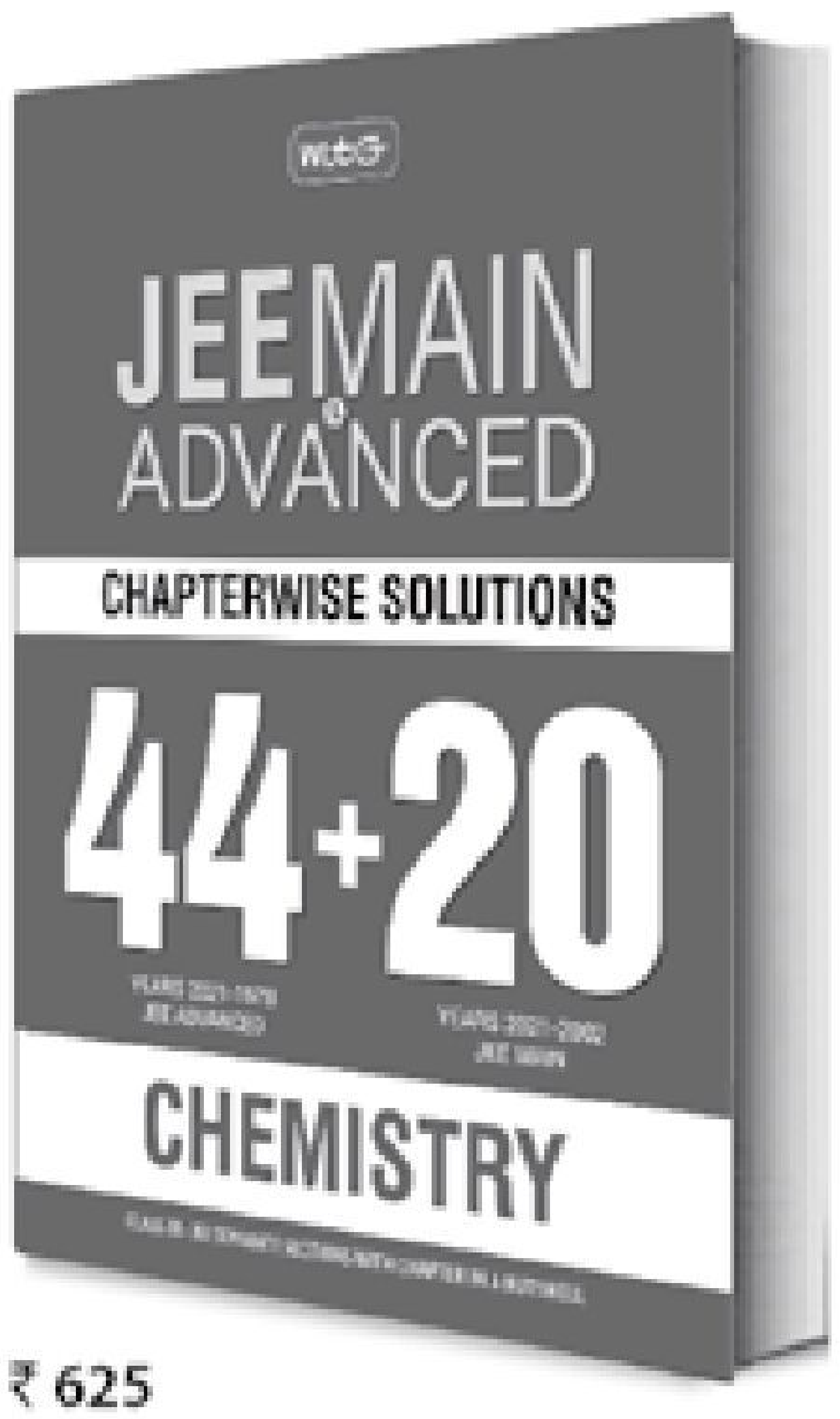
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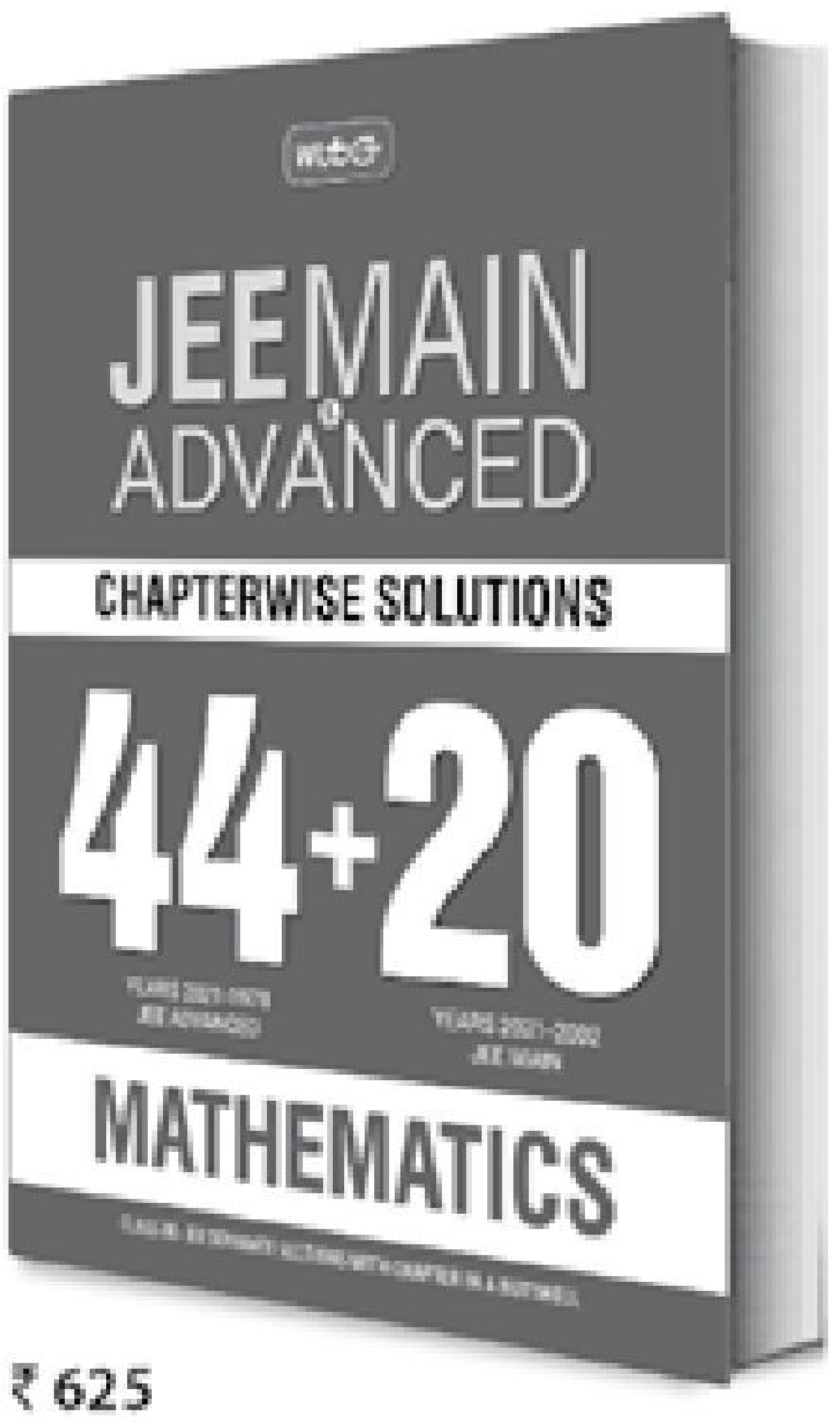
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But since there are only ten w_i's this is impossible. Thus $(M, m) = (5, 4)$, and $(4, 4, 4, 4, 4, 5, 5, 5, 5, 5)$ is the only possible solution. By exhibiting the tournament graphically, one can see that this is indeed a solution.

Now, there is a theorem in graph theory which states that the number of transitive triples [3 players P_i, P_j and P_k such that P_i beats P_j, P_j beats P_k, and P_i beats P_k – *Ed.*] in a tournament with n vertices and score sequence $(w_1, w_2, \dots, w_n)$ is $\sum_{i=1}^{n} w_i (w_i - 1)/2$.

Note : This result is true because the number of such triples is just the number of unordered pairs $\{P_j, P_k\}$ of (distinct) players beaten by P_i, which is $\binom{w_i}{2}$, added up over all P_i.] It follows that, with $n = 10$, the number of triangles (cyclic triples) is

$$\binom{10}{3} - \frac{1}{2}\sum_{i=1}^{n} w_i(w_i - 1)$$

$$= 120 - \frac{1}{2}(5)\ (5)(5-1) - \frac{1}{2}(5)(4)(4-1) = 40.$$

This completes the proof.

5. For all real x, let $\{x\} = x - [x] \in [0, 1)$.

Writing $\left\lfloor \frac{2k^2}{p} \right\rfloor = \frac{2k^2}{p} - \left\{\frac{2k^2}{p}\right\}$ and $\left\lfloor \frac{k^2}{p} \right\rfloor = \frac{k^2}{p} - \left\{\frac{k^2}{p}\right\}$,

we find that $\left\lfloor \frac{2k^2}{p} \right\rfloor - 2\left\lfloor \frac{k^2}{p} \right\rfloor = 2\left\{\frac{k^2}{p}\right\} - \left\{\frac{2k^2}{p}\right\}$.

When $\{x\} < \frac{1}{2}, 2\{x\} - 2\{x\} = 2\{x\} - 2\{x\} = 0$.

When $\{x\} \ge \frac{1}{2}, 2\{x\} - \{2x\} = 2\{x\} - (2\{x\} - 1) = 1$.

Therefore, the desired sum equals the number $\propto$ of k in $[1, p - 1]$ such that $\left\{\frac{k^2}{p}\right\} \ge \frac{1}{2}$, or equivalently, the number of nonzero residues k modulo p such that k^2 is congruent to some number in $\left[\frac{(p+1)}{2}, p-1\right]$ modulo p.

Because p is a prime congruent to 1 modulo 4, it is well known that $-1 \equiv d^2 \pmod{p}$ for some integer d. Partition the nonzero residues modulo p into $\frac{p-1}{2}$ pairs of the form $\{a, da\}$ so that $a^2 \equiv -(da)^2 \pmod{p}$ in each pair. Thus, exactly one residue in each pair has a square congruent to some number in $\left[\frac{p+1}{2}, p-1\right]$, for a total of $\frac{p-1}{2}$ such residues. It follows that the given sum equals $\frac{p-1}{2}$.

6. Equal chords subtend equal angles at the centre of a circle; if each of sides of length i subtends an angle α_i $(i = 1, 2, 3)$ at the centre of the given circle, then $2\alpha_1 + 2\alpha_2 + 2\alpha_3 = 360°$,

where, $\frac{\alpha_1}{2} + \frac{\alpha_2}{2} = 90° - \frac{\alpha_3}{2}$,

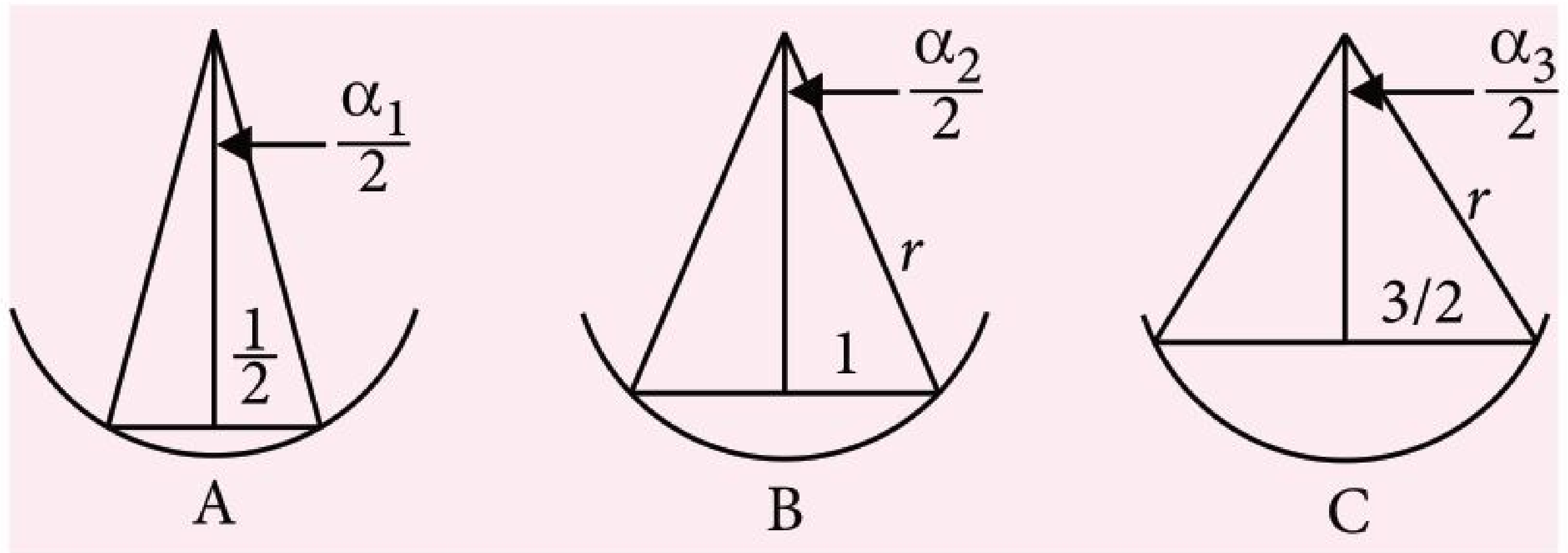
and $\cos\left(\frac{\alpha_1}{2} + \frac{\alpha_2}{2}\right) = \cos\left(90° - \frac{\alpha_3}{2}\right) = \sin\frac{\alpha_3}{2}$,

Next we apply the addition formula for the cosine:

$$\cos\frac{\alpha_1}{2}\cos\frac{\alpha_2}{2} - \sin\frac{\alpha_1}{2}\sin\frac{\alpha_2}{2} = \sin\frac{\alpha_3}{2}, \qquad \dots\text{(i)}$$

where, $\sin\frac{\alpha_1}{2} = \frac{1/2}{r}$ $\cos\frac{\alpha_1}{2} = \frac{\sqrt{4r^2 - 1}}{2r}$;

$\sin\frac{\alpha_2}{2} = \frac{1}{r}$, $\cos\frac{\alpha_2}{2} = \frac{\sqrt{r^2 - 1}}{r}$; $\sin\frac{\alpha_3}{2} = \frac{3/2}{r}$.



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We substitute these expressions into (i) and obtain, after multiplying both sides by $2r^2$,

$$\sqrt{4r^2-1}.\sqrt{r^2-1}-1=3r.$$

Now write it in the form $\sqrt{(4r^2-1)(r^2-1)}=3r+1$, and square, obtaining $(4r^2 - 1)(r^2 - 1) = 9r^2 + 6r + 1$,

which is equivalent to $r(2r^3 - 7r - 3) = 0$.

Since $r \neq 0$, we have $2r^3 - 7r - 3 = 0$, which was to be shown.

7. Since, $1^3+2^3+\ldots+n^3=\left(\frac{n(n+1)}{2}\right)^2$

we see that when $k = 0$,

$$(x_1,x_2,\ldots,x_n;y)=\left(1,2,\ldots,n;\frac{n(n+1)}{2}\right)$$

is a solution. To see that we can generate infinitely many solutions. In general, set $c = (n(n + 1))/2$ and notice that for all positive integers q, we have:

$(c^kq^{3k+2})^3 + (2c^kq^{3k+2})^3 + \ldots + (nc^kq^{3k+2})^3$
$= c^{3k}q^{3(3k+2)}\,(1^3 + 2^3 + \ldots + n^3)$
$= c^{3k}\,q^{3(3k+2)}\left(\frac{n(n+1)}{2}\right)^2$
$= c^{3k+2}q^{3(3k+2)} = (cq^3)^{3k+2}$.

That is, $(x_1, x_2,\ldots,x_n;y) = (c^kq^{3k+2}, 2c^kq^{3k+2}, \ldots, nc^kq^{3k+2}; cq^3)$ is a solution. This completes the proof.

8. A binary string is a finite string of digits, all either 0 or 1. Call such a string (perhaps starting with zeroes) valid if it does not contain three consecutive equal digits. Let a_n represent the number of valid n-digit strings; let s_n be the number of valid strings starting with two equal digits; and let d_n be the number of valid strings starting with two different digits. Observe that $a_n = s_n + d_n$ for all n.

An $(n + 2)$-digit string starting with 00 is valid if and only if its last n digits form a valid string starting with 1. Similarly, an $(n + 2)$-digit string starting with 11 is valid if and only if its last n digits form a valid string starting with 0. Thus, $S_{n+2} = a_n = s_n + d_n$.

An $(n + 2)$-digit string starting with 01 is valid if and only if its last n digits form a valid string starting with 00, 01, or 10. Similarly, an $(n + 2)$-digit string starting with 10 is valid if and only if its last n digits form a valid string starting with 11, 01, or 10. Thus, $d_{n+2} = s_n + 2d_n$. Solving these recursions gives $s_{n+4} = 3s_{n+2} - s_n$ and $d_{n+4} = 3d_{n+2} - d_n$, which when added together yield

$a_{n+4} = 3a_{n+2} - a_n$.

Thus we can calculate initial values of a_n and then use the recursion to find other values.

n	1	2	3	4	5	6	7	8	9	10
a_n	2	4	6	10	16	26	42	68	110	178

Now of the a_n valid n-digit strings, only half start with 1. Thus only half are binary representations of positive numbers, and exactly

$$\frac{1}{2}(a_1+a_2+.......+a_{10})=231$$

numbers between 1 and 1023 have the desired property. Ignoring 1,2, and 3, we find that the answer is 228.

◆◆

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

			4			9									2					1
	8	3											9				8			
	9	4	2		7	6								2			9			4
			9			8	7						5		9					7
								5						8	5		6		2	
7	6				3			2								3	2		4	
	2	5			8	1			5				3			7				
			6	4			8				1	7			4			3		
		8		1		7							1	9			3			6
									6											
								6			2		5	8						
							9		3	1			6							
	7				6	4		2								4		7		2
	9		7		5		3		9	5					9		5			
1				3						6	3								8	
		2		4	1			5					5			6				3
		8										4		9						1
			5			2	9					3	2						4	
4		6						7						2			8	6		
					2	5		6						4		9				7
					3			9					3				6			5

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

YOU ASK
WE ANSWER

Do you have a question that you just can't get answered? Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. A man parks his car among n cars standing in a row, his car not being parked at an end. On his return he finds that exactly m of the n cars are still there. What is the probability that both the cars parked on two sides of his car have left? ***[Ramesh, Ludhina]***

Ans. Clearly, his car is at one of the crosses (×).

$$|\times\times\times\cdots\times\times\times|$$

The number of the ways in which the remaining $(m - 1)$ cars can take their places (excluding the car of the man) $= {}^{n-1}C_{m-1}$ {∵ there are $(n - 1)$ places for the $(m - 1)$ cars}

The number of ways in which the remaining $(m - 1)$ cars can take places keeping the two places on two sides of his car $= {}^{n-3}C_{m-1}$

$$\therefore \quad \text{required probability } = \frac{n(E)}{n(S)} = \frac{{}^{n-3}C_{m-1}}{{}^{n-1}C_{m-1}}$$

$$= \frac{(n-3)!}{(m-1)!(n-m-2)!} \times \frac{(m-1)!(n-m)!}{(n-1)!} = \frac{(n-m)(n-m-1)}{(n-1)(n-2)}.$$

2. If $\tan^{-1}y : \tan^{-1}x = 4 : 1$, express y as an algebraic function of x. Hence or otherwise prove that $\tan 22\frac{1^\circ}{2}$ is a root of the equation $x^4 + 1 = 6x^2$. ***[Rahil, U.P.]***

Ans. Here, $\tan^{-1}y = 4\tan^{-1}x = 2(2\tan^{-1}x)$

$$= 2\tan^{-1}\frac{2x}{1-x^2}$$

$$= \tan^{-1}\frac{2\cdot\dfrac{2x}{1-x^2}}{1-\left(\dfrac{2x}{1-x^2}\right)^2} = \tan^{-1}\frac{4x(1-x^2)}{(1-x^2)^2-4x^2}$$

$$\text{or} \quad y = \frac{4x(1-x^2)}{1+x^4-6x^2}$$

Let $\tan^{-1}x = 22\frac{1^\circ}{2}$; then $4\tan^{-1}x = 90^\circ$

$$\Rightarrow \quad \tan^{-1}y = 90^\circ \ \{\because \tan^{-1}y = 4\tan^{-1}x\}$$

$$\Rightarrow \quad y = \tan 90^\circ = \frac{4x(1-x^2)}{1+x^4-6x^2}$$

$$\Rightarrow \quad 1 + x^4 - 6x^2 = 0 \Rightarrow 1 + x^4 = 6x^2$$

$\therefore \quad x = \tan 22\frac{1^\circ}{2}$ is a root of the equation $1 + x^4 = 6x^2$.

3. If for $0 < x < \pi/2$, $\exp[(\sin^2x + \sin^4x + \sin^6x + \ldots\infty)\log_e 2]$ satisfies the quadratic equation $x^2 - 9x + 8 = 0$, find the value of $\dfrac{\sin x - \cos x}{\sin x + \cos x}$. ***[Nalini, Bangalore]***

Ans. $\because\ 0 < x < \pi/2 \ \therefore\ 0 < \sin^2x < 1$

Then $\sin^2x + \sin^4x + \sin^6x + \ldots\infty = \dfrac{\sin^2 x}{1-\sin^2 x} = \tan^2x$

$$\therefore \quad \exp[(\sin^2x + \sin^4x + \sin^6x + \ldots\infty)\log_e 2]$$
$$= \exp\{\tan^2x \log_e 2\}$$
$$= \exp\{\log_e 2^{\tan^2 x}\} = e^{\log_e 2^{\tan^2 x}} = 2^{\tan^2 x}$$

Let $y = 2^{\tan^2 x}$

$\because \quad y$ satisfies the quadratic equation

$y^2 - 9y + 8 = 0 \ \therefore\ y = 1, 8$

if $y = 1 = 2^{\tan^2 x}$ then $2^{\tan^2 x} = 2^0$

$\therefore \quad \tan^2x = 0$

$\therefore \quad x = 0$ (impossible) $\quad (\because x > 0)$

Now if $y = 8 = 2^{\tan^2 x}$ then $2^{\tan^2 x} = 2^3$

$\Rightarrow \quad \tan^2x = 3 \ \therefore\ \tan x = \sqrt{3}$

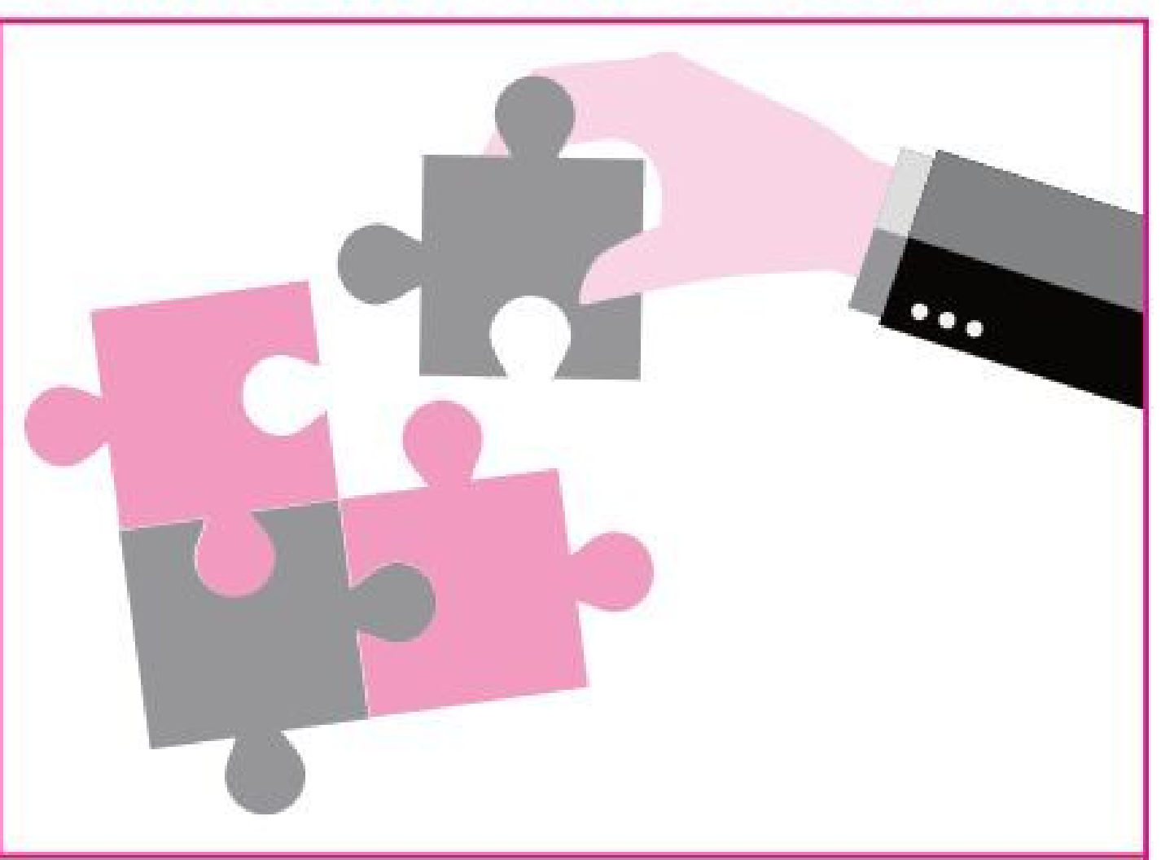
$$\therefore \quad \frac{\sin x - \cos x}{\sin x + \cos x} = \frac{\tan x - 1}{\tan x + 1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2}{3-1} = \frac{3+1-2\sqrt{3}}{2} = 2-\sqrt{3}.$$

◆ ◆

PUZZLE
CORNER

SOLUTION - MARCH 2022



3 3	1– 5	6× 1	4– 6	2	1– 4
1– 4	6	2	3+ 1	3 3	5
5	1 1	3	2	18+ 4	6
15× 1	3	5	20× 4	6	2
3÷ 2	32× 4	6 6	5	15× 1	4+ 3
6	2	4	3	5	1

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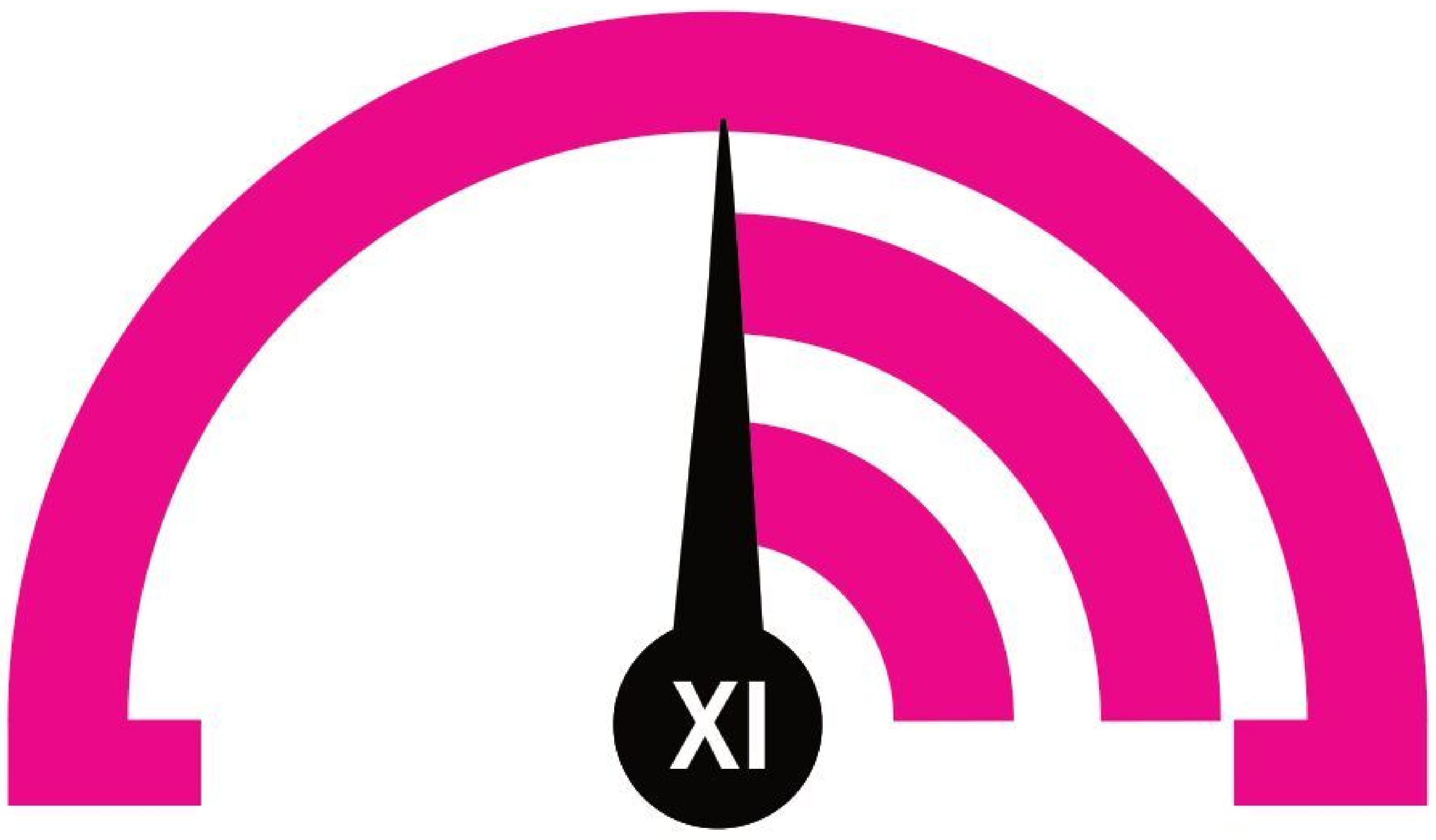

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. If $f(x)=\frac{ax}{x+1}, x\neq -1$ then for what value of a is $f(f(x))=x$?

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 1 (d) –1

2. The product of perpendicular drawn from any point on $\frac{x^2}{9}-\frac{y^2}{16}=1$ upon its asymptote is

(a) $\frac{125}{144}$ (b) $\frac{144}{25}$ (c) $\frac{25}{144}$ (d) $\frac{144}{125}$

3. The domain of the function

$$f(x)=\log_2\left(-\log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right) \text{ is}$$

(a) (0, 1) (b) (0, 1]
(c) [1, ∞) (d) (1, ∞)

4. The sum (in radians) of all values of x with $0\le x\le 2\pi$ which satisfy $\sqrt{2}$ $(\cos 2x - \sin x - 1) = 1 + 2\sin x$, is

(a) 2π (b) 3π (c) 4π (d) 6π

5. $$\lim_{x\to 1}\left[\left[\frac{4}{x^2-x^{-1}}-\frac{1-3x+x^2}{1-x^3}\right]^{-1}+\frac{3(x^4-1)}{x^3-x^{-1}}\right]=$$

(a) 1/3 (b) 3
(d) 1/2 (d) none of these

6. $\sum_{n=1}^{\infty}\frac{n}{4n^4+1}$ equals to

(a) 0 (b) 1
(c) ∞ (d) none of these

One or More Than One Option(s) Correct Type

7. The variable x takes two values x_1 and x_2 with frequencies f_1 and f_2, respectively. If σ denotes the standard deviation of x, then

(a) $\sigma^2=\frac{f_1x_1^2+f_2x_2^2}{f_1+f_2}-\left(\frac{f_1x_1+f_2x_2}{f_1+f_2}\right)^2$

(b) $\sigma^2=\frac{f_1f_2}{(f_1+f_2)^2}(x_1-x_2)^2$

(c) $\sigma^2=\frac{(x_1-x_2)^2}{(f_1+f_2)^2}$

(d) none of these

8. The equation of a common tangent to the parabola $y^2 = 2x$ and the circle $x^2 + y^2 + 4x = 0$ is

(a) $2\sqrt{6}x+y=12$ (b) $x+2\sqrt{6}y+12=0$
(c) $x-2\sqrt{6}y+12=0$ (d) $2\sqrt{6}x-y=12$

9. The chance of an event happening is the square of the chance of a second event but the odds against the first are the cube of the odds against the seconds. The chances of the events are

(a) $p_1=\frac{1}{9}$ (b) $p_1=\frac{1}{16}$
(c) $p_2=\frac{1}{3}$ (d) $p_2=\frac{1}{4}$

10. If $y=x^{(\ln x)^{\ln(\ln x)}}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{y}{x}(\ln x^{(\ln x-1)}+2(\ln x)\ln(\ln x))$

(b) $\frac{y}{x}(\ln x)^{\ln(\ln x)}(2\ln(\ln x)+1)$

(c) $\dfrac{y}{x\ln x}((\ln x)^2 + 2\ln(\ln x))$

(d) $\dfrac{y\ln y}{x\ln x}(2\ln(\ln x)+1)$

11. The coefficient of x^n in the expansion of $(1 - 9x + 20x^2)^{-1}$ is

(a) $5^n - 4^n$ (b) $5^{n+1} - 4^{n+1}$

(c) $5^{n-1} - 4^{n-1}$ (d) none of these

12. Which of the following set satisfy the equation $\frac{\sqrt{3}}{2}\sin x - \cos x = \cos^2 x$

(a) $x = (2n+1)\pi$ (b) $x = 2n\pi \pm \frac{\pi}{3}$

(c) $x = 2n\pi \pm \frac{\pi}{6}$ (d) none of these

Comprehension Type

Paragraph for Q. No. 13 to 15

In a triangle OAB, the sides OA and OB are given by $y = mx$ and $y = -mx$ respectively.

13. If the orthocentre is $H(a, b)$, then the equation of AB is

(a) $(1 + m^2)(ax + by) = a^2 + b^2m^2$

(b) $(1 + m^2)(ax - by) = a^2 - b^2m^2$

(c) $(1 - m^2)(ax + by) = a^2 + b^2m^2$

(d) $(1 - m^2)(ax + by) = a^2 - b^2m^2$

14. If the circumcentre is $S(a, b)$, then the equation of AB is

(a) $(1 + m^2)(ax + by) = 2(a^2 + b^2m^2)$

(b) $(1 + m^2)(ax - by) = 2(a^2 - b^2m^2)$

(c) $(1 - m^2)(ax + by) = 2(a^2 + b^2m^2)$

(d) $(1 - m^2)(ax - by) = 2(a^2 - b^2m^2)$

15. If the centroid is $G(a, b)$, then the equation of AB is

(a) $2(am^2x - by) = 3(a^2m^2 - b^2)$

(b) $2(ax - bm^2y) = 3(a^2 - b^2m^2)$

(c) $3(am^2x - by) = 2(a^2m^2 - b^2)$

(d) $3(am^2x + by) = 2(a^2m^2 + b^2)$

Matrix Match Type

16. Match the following :

	Column-I	Column-II	
P.	Let x be the arithmetic mean and y, z be two geometric means between any two positive numbers, then $(y^3 + z^3) / xyz$ is equal	1.	1
Q.	The sum $\frac{2}{4-1} + \frac{2^2}{4^2-1} + \frac{2^4}{4^4-1} + \ldots\ldots\infty$ is	2.	2
R.	If $S_n = \sum_{n=1}^{n} \frac{2n+1}{n^4 + 2n^3 + n^2}$, then $S_{10} <$	3.	3
S.	If a, b, c be three positive number which form three successive terms of a G. P. and $c > 4b - 3a$, then the common ratio of the G. P. can be equal to	4.	4
		5.	5

	P	Q	R	S
(a)	3, 4	5	1, 2, 3	4, 5
(b)	2	1, 2, 3	4, 5	1, 2
(c)	2	1	1, 2, 3, 4, 5	4, 5
(d)	3, 5	1, 2, 3, 4	4, 5	1, 2, 3

Numerical Answer Type

17. If l_1 and l_2 be the length of the focal chord other than latus rectum of ellipse $x^2/4 + y^2/3 = 1$ which are perpendicular to each other, than $12\,(1/l_1 + 1/l_2) =$

18. The number of real roots of the equation $(15 + 4\sqrt{14})^t + (15 - 4\sqrt{14})^t = 30$, where $t = x^2 - 4|x|$ is

19. If $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, then the last digit of n is

20. In how many ways five different rings can be worn in four fingers with at least one ring in each finger?

◆ ◆

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SELF CHECK

No. of questions attempted ……

No. of questions correct ……

Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

CBSE warm-up!

CLASS-XII

Term-II
Exam on
7th June

Practice Paper 2022

Time allowed : 2 Hours
Maximum marks : 40

General Instructions :

1. *This question paper contains three sections - A, B and C. Each part is compulsory.*
2. *Section - A has 6 short answer type (SA1) questions of 2 marks each.*
3. *Section - B has 4 short answer type (SA2) questions of 3 marks each.*
4. *Section - C has 4 long answer type questions (LA) of 4 marks each.*
5. *There is an internal choice in some of the questions.*
6. *Q14 is a case-based problem having 2 sub parts of 2 marks each.*

SECTION - A

1. If α, β, γ are the angles made by a line with the co-ordinate axes. Then, find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

2. Find the order of the differential equation of all tangent lines to the parabola $y = x^2$.

OR

Write the sum of the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$.

3. Given, $\vec{p} = 3\hat{i} + 2\hat{j} + 4\hat{k}, \vec{a} = \hat{i} + \hat{j}, b = \hat{j} + \hat{k}, \vec{c} = \hat{i} + \hat{k}$ and $\vec{p} = x\vec{a} + y\vec{b} + z\vec{c}$, then find the value of x, y, z.

4. Find : $\int \frac{dx}{x^2 + 4x + 8}$

5. A problem is given to three students whose probabilities of solving it are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{6}$ respectively. If the events of solving the problem are independent, find the probability that at least one of them solves it.

6. An unbiased dice is thrown twice. Let the event A be 'odd number on the first throw' and B be the event 'odd number on the second throw'. Check the independence of the events A and B.

SECTION - B

7. Evaluate : $\int_{\pi/5}^{3\pi/10} \frac{\sin x}{(\sin x + \cos x)} dx$

OR

Evaluate : $\int \frac{3x + 1}{(x+1)^2(x+3)} dx$

8. Show that the differential equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ is homogeneous and also solve it.

9. Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{k}$.

10. Find the value of λ, so that the following lines are perpendicular to each other :

$$\frac{x-5}{7}=\frac{2-y}{\lambda}=\frac{1-z}{-1} \text{ and } \frac{x}{\lambda}=\frac{2y+1}{4}=\frac{z-1}{3}$$

OR

A plane makes intercepts 6, –3, –4 respectively on the coordinate axes. Find the length of the perpendicular from the origin on it.

SECTION - C

11. If $\vec{a}=4\hat{i}+5\hat{j}-\hat{k}, \vec{b}=\hat{i}-4\hat{j}+5\hat{k}, \vec{c}=3\hat{i}+\hat{j}-\hat{k}$, then find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}\cdot\vec{d}=21$.

12. Find the area bounded by the lines $y = ||x| - 1|$ and the x-axis.

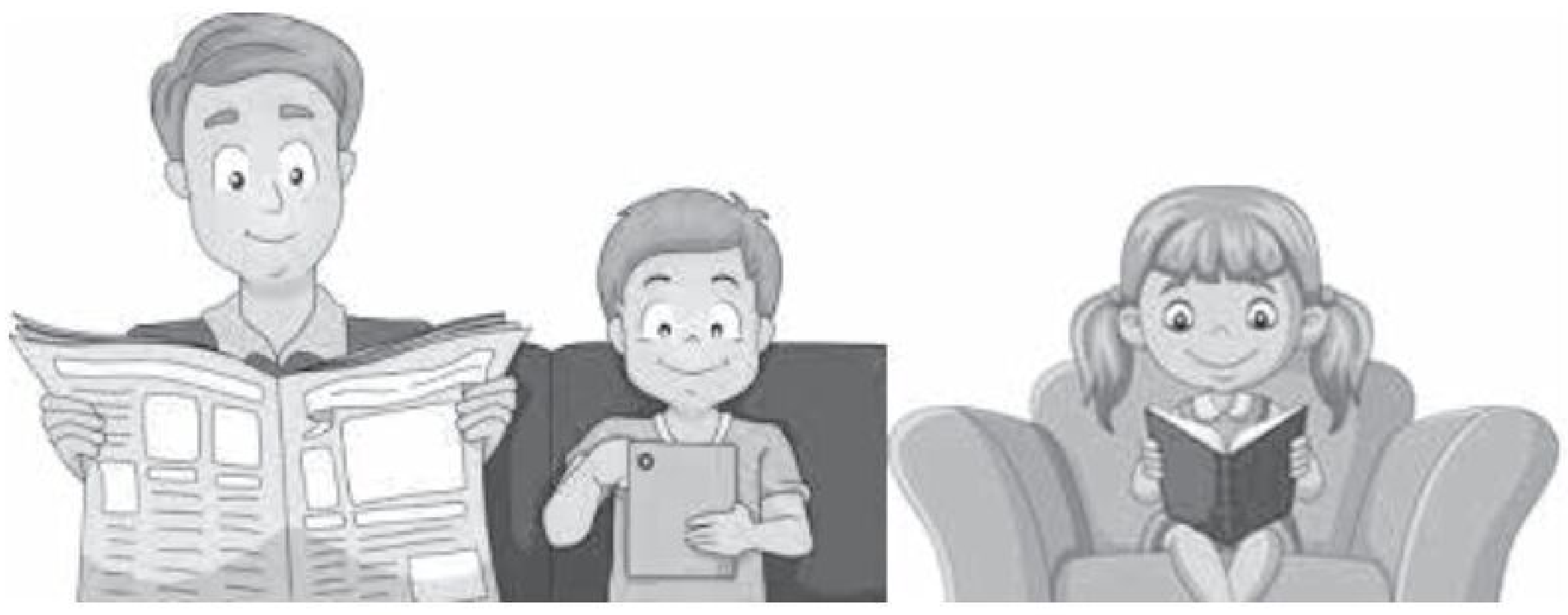
13. Evaluate : $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$

OR

Evaluate : $\int\frac{x^2+1}{(x-1)^2(x+3)}dx$

CASE-BASED / DATA-BASED

14. On a holiday, a father gave a puzzle from a newspaper to his son Ravi and his daughter Priya. The probability of solving this specific puzzle independently by Ravi and Priya are $\frac{1}{4}$ and $\frac{1}{5}$ respectively.



Based on the above information, answer the following questions.

(i) Find the probability that puzzle is solved.

(ii) Find the probability that exactly one of them solved the puzzle.

SOLUTIONS

1. Since, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow$ $1-\sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow$ $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

2. The parametric form of the given equation is $x = t, y = t^2$.

The equation of any tangent at t is $2xt = y + t^2$

On differentiating, we get $2t = y_1$

On putting this value in the above equation, we get

$$xy_1 = y+\left(\frac{y_1}{2}\right)^2 \Rightarrow 4xy_1 = 4y + y_1^2$$

The order of this equation is 1.

OR

Order = 2, Degree = 2.

$\therefore$ Required sum = 2 + 2 = 4

3. We have, $\vec{p}=x\vec{a}+y\vec{b}+z\vec{c}$

$\Rightarrow$ $3\hat{i}+2\hat{j}+4\hat{k}=x(\hat{i}+\hat{j})+y(\hat{j}+\hat{k})+z(\hat{i}+\hat{k})$

$\Rightarrow$ $3\hat{i}+2\hat{j}+4\hat{k}=(x+z)\hat{i}+(x+y)\hat{j}+(y+z)\hat{k}$

$\Rightarrow$ $x + z = 3$ ...(i), $x + y = 2$ ...(ii)

and $y + z = 4$ ...(iii)

On solving (i), (ii) and (iii), we get $x=\frac{1}{2}, y=\frac{3}{2}, z=\frac{5}{2}$

4. We have, $\int\frac{dx}{x^2+4x+8}=\int\frac{dx}{x^2+4x+4+4}$

$$=\int\frac{dx}{(x+2)^2+(2)^2}=\frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right)+C$$

5. Let A, B, C be respectively the events of solving problem by three students and $P(A)$, $P(B)$, $P(C)$ be their probability of solving the problem respectively.

$\therefore$ $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{4}$ and $P(C)=\frac{1}{6}$

Required probability = $1 - P(\bar{A}\cap\bar{B}\cap\bar{C})$

= $1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C})$

($\because$ A, B, C are independent $\therefore$ $\bar{A}$, $\bar{B}$, $\bar{C}$ are also independent)

= $1 - [1 - P(A)]\,[1 - P(B)]\,[1 - P(C)]$

$$=1-\frac{2}{3}\times\frac{3}{4}\times\frac{5}{6}=1-\frac{5}{12}=\frac{7}{12}$$

6. If all the 36 elementary events of the experiment are considered to be equally likely, we have

$$P(A)=\frac{18}{36}=\frac{1}{2} \text{ and } P(B)=\frac{18}{36}=\frac{1}{2}$$

Also, $P(A\cap B) = P$(odd number on both throws) $=\frac{9}{36}=\frac{1}{4}$

Now, $P(A)P(B)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

Clearly $P(A\cap B) = P(A)\times P(B)$

Thus, A and B are independent events.

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7. Let $I=\int_{\pi/5}^{3\pi/10}\frac{\sin x}{(\sin x+\cos x)}dx$ ...(i)

Here, $a=\frac{\pi}{5}$ and $b=\frac{3\pi}{10} \Rightarrow (a+b)=\left(\frac{\pi}{5}+\frac{3\pi}{10}\right)=\frac{\pi}{2}$

Using $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx,$ we have

$$I=\int_{\pi/5}^{3\pi/10}\frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$\Rightarrow \quad I=\int_{\pi/5}^{3\pi/10}\frac{\cos x}{\cos x+\sin x}dx$ ...(ii)

Adding (i) and (ii), we get

$$2I=\int_{\pi/5}^{3\pi/10} 1dx=[x]_{\pi/5}^{3\pi/10}=\left(\frac{3\pi}{10}-\frac{\pi}{5}\right)=\frac{\pi}{10} \Rightarrow I=\frac{\pi}{20}$$

OR

Let $I=\int\frac{3x+1}{(x+1)^2(x+3)}dx$

Let $\frac{3x+1}{(x+1)^2(x+3)}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x+3}$ ...(1)

$\Rightarrow \quad 3x+1=A(x+1)(x+3)+B(x+3)+C(x+1)^2$ ...(2)

Put $x=-1, -3, 0$ in (2), we get

$B=-1; C=-2; A=2$

$\therefore \quad$ From (1), $\frac{3x+1}{(x+1)^2(x+3)}=\frac{2}{x+1}-\frac{1}{(x+1)^2}-\frac{2}{x+3}$

Integrating both sides w.r.t. x, we get

$$I=\int\frac{2}{x+1}dx-\int\frac{1}{(x+1)^2}dx-\int\frac{2}{(x+3)}dx$$

$$=2\log|x+1|+\frac{1}{x+1}-2\log|x+3|+C_1$$

$$=2\log\left|\frac{x+1}{x+3}\right|+\frac{1}{x+1}+C_1$$

8. We have, $\frac{dy}{dx}=\frac{y^2}{xy-x^2}=\frac{y^2/x^2}{(xy-x^2)/x^2}$ ...(i)

$\Rightarrow \quad \frac{dy}{dx}=\frac{y^2/x^2}{\frac{y}{x}-1}$

$\Rightarrow$ It is a homogeneous differential equation.

$\therefore \quad$ Put $y=vx \Rightarrow \frac{dy}{dx}=v\cdot 1+x\frac{dv}{dx}$

From (i),

$$v+x\frac{dv}{dx}=\frac{v^2}{v-1} \Rightarrow x\frac{dv}{dx}=\frac{v^2}{v-1}-v \Rightarrow x\frac{dv}{dx}=\frac{v}{v-1}$$

$\Rightarrow \quad \frac{v-1}{v}dv=\frac{dx}{x} \Rightarrow \left(1-\frac{1}{v}\right)dv=\frac{dx}{x}$

Integrating both sides, we get

$v-\log v=\log x+C \Rightarrow v=\log vx+C$

$\frac{y}{x}=\log y+C$

$\Rightarrow \quad y=x(\log y+C)$ is the required solution.

9. We are given, $\vec{a}=\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{b}=-\hat{i}+3\hat{k}$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 3 & -2\\ -1 & 0 & 3\end{vmatrix}$

$$=(9-0)\hat{i}-(3-2)\hat{j}+(0+3)\hat{k}=9\hat{i}-\hat{j}+3\hat{k}$$

$\therefore \quad |\vec{a}\times\vec{b}|=\sqrt{9^2+(-1)^2+3^2}=\sqrt{81+1+9}=\sqrt{91}$

10. We have, $\frac{x-5}{7}=\frac{2-y}{\lambda}=\frac{1-z}{-1}$ and $\frac{x}{\lambda}=\frac{2y+1}{4}=\frac{z-1}{3}$

These equations may be written in standard form as

$$\frac{x-5}{7}=\frac{y-2}{-\lambda}=\frac{z-1}{1} \text{ and } \frac{x}{\lambda}=\frac{y+1/2}{2}=\frac{z-1}{3}$$

Since, the lines are perpendicular, so

$7(\lambda)+(-\lambda)2+(1)(3)=0 \Rightarrow 7\lambda-2\lambda+3=0$

$\Rightarrow \quad 5\lambda+3=0 \quad \Rightarrow \lambda=-3/5$

OR

Given that 6, -3, -4 are intercepts on x, y and z-axes respectively.

$\therefore \quad$ Equation of the plane is $\frac{x}{6}+\frac{y}{-3}+\frac{z}{-4}=1$

$\Rightarrow \quad 2x-4y-3z-12=0$

$\therefore \quad$ Length of the perpendicular from origin to the plane $2x-4y-3z-12=0$ is

$$\left|\frac{2\times0-4\times0-3\times0-12}{\sqrt{(2)^2+(-4)^2+(-3)^2}}\right|=\frac{12}{\sqrt{29}} \text{ units}$$

11. We have,

$$\vec{a}=4\hat{i}+5\hat{j}-\hat{k},\ \vec{b}=\hat{i}-4\hat{j}+5\hat{k},\ \vec{c}=3\hat{i}+\hat{j}-\hat{k}$$

Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, therefore it is parallel to $\vec{a}\times\vec{b}$. So, let $\vec{d}=\lambda(\vec{a}\times\vec{b})$

$\Rightarrow \quad \vec{d}=\lambda\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 5 & -1\\ 1 & -4 & 5\end{vmatrix}$

$$=\lambda[(25-4)\hat{i}-(20+1)\hat{j}+(-16-5)\hat{k}]$$

$\Rightarrow \quad \vec{d}=\lambda(21\hat{i}-21\hat{j}-21\hat{k})$ ...(i)

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Now, $\vec{c}\cdot\vec{d}=21$

$\Rightarrow (3\hat{i}+\hat{j}-\hat{k})\cdot\lambda(21\hat{i}-21\hat{j}-21\hat{k})=21$

$\Rightarrow 21\lambda(3-1+1)=21 \Rightarrow \lambda=\frac{1}{3}$

Putting $\lambda=\frac{1}{3}$ in (i), we get

$\vec{d}=\frac{1}{3}(21\hat{i}-21\hat{j}-21\hat{k})=7\hat{i}-7\hat{j}-7\hat{k}$

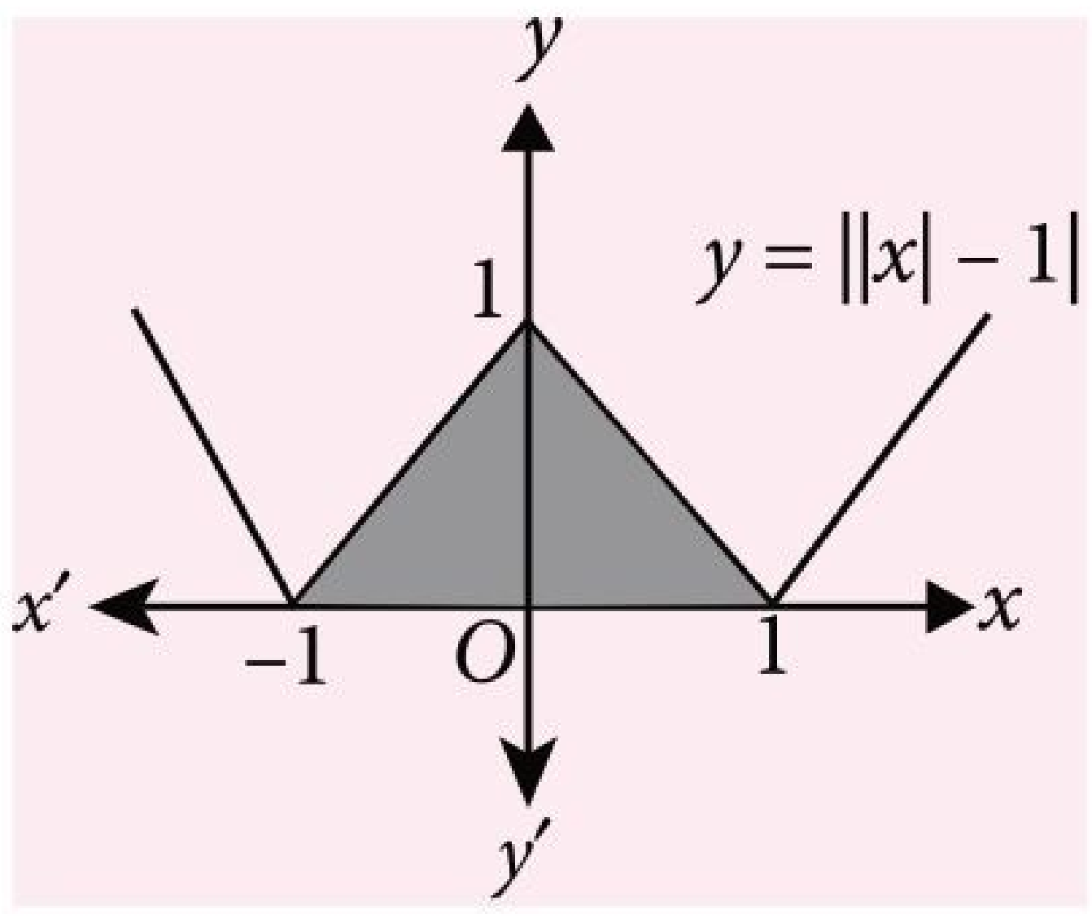
12. We have, $y=|x-1|$, if $x\geq 0$

$y=\begin{cases}(x-1), & \text{if } x\geq 1\\ -(x-1), & \text{if } x<1\end{cases}$ and

$y=|-x-1|=|-(x+1)|$

$=|x+1|$, if $x<0$

$y=\begin{cases}(x+1), & \text{if } x\geq -1\\ -(x+1), & \text{if } x<-1\end{cases}$



$\therefore$ Required area $=2\int_0^1(1-x)\,dx$

$=2\left[x-\frac{x^2}{2}\right]_0^1=2\times\frac{1}{2}=1$ sq. unit

13. Let $I=\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$ ... (i)

$=\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+[\cos(\pi-x)]^2}dx \quad \left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$

$\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx$ ...(ii)

Adding (i) and (ii), we get

$$2I=\int_0^{\pi}\frac{\pi\sin x}{1+\cos^2 x}dx$$

Put $\cos x=t \Rightarrow -\sin x\,dx=dt$

Also, $x=0 \Rightarrow t=1$ and $x=\pi \Rightarrow t=-1$

$\therefore 2I=\int_1^{-1}\frac{-\pi dt}{1+t^2} \Rightarrow I=\frac{\pi}{2}\int_{-1}^{1}\frac{dt}{1+t^2}$

$\therefore I=\frac{\pi}{2}[\tan^{-1}t]_{-1}^{1}=\frac{\pi}{2}[\tan^{-1}(1)-\tan^{-1}(-1)]$

$=\frac{\pi}{2}\left[\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)\right]=\frac{\pi^2}{4}$

OR

Let $I=\int\frac{x^2+1}{(x-1)^2(x+3)}dx$

Let $\frac{x^2+1}{(x-1)^2(x+3)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+3}$

$\Rightarrow x^2+1=A(x-1)(x+3)+B(x+3)+C(x-1)^2$ ...(1)

Put $x=1$ in (1), we get $B=\frac{1}{2}$

Put $x=-3$ in (1), we get $C=\frac{5}{8}$

Put $x=0$ in (1), we get $A=\frac{3}{8}$

$\therefore \frac{x^2+1}{(x-1)^2(x+3)}=\frac{3}{8}\cdot\frac{1}{x-1}+\frac{1}{2}\cdot\frac{1}{(x-1)^2}+\frac{5}{8}\cdot\frac{1}{x+3}$

Integrating both sides, we get

$I=\int\frac{x^2+1}{(x-1)^2(x+3)}dx=\frac{3}{8}\int\frac{dx}{(x-1)}+\frac{1}{2}\int\frac{dx}{(x-1)^2}+\frac{5}{8}\int\frac{dx}{x+3}$

$=\frac{3}{8}\log|x-1|-\frac{1}{2}\cdot\frac{1}{(x-1)}+\frac{5}{8}\log|x+3|+C_1$

14. Let E_1 be the event that Ravi solved the puzzle and E_2 be the event that Priya solved the puzzle. Then, $P(E_1)=\frac{1}{4}$ and $P(E_2)=\frac{1}{5}$

(i) P(puzzle is solved) $=P(E_1$ or $E_2)$

$=P(E_1\cup E_2)=P(E_1)+P(E_2)-P(E_1\cap E_2)$

$=\frac{1}{4}+\frac{1}{5}-\frac{1}{20}=\frac{8}{20}=\frac{2}{5}$

(ii) P(Exactly one of them solved the puzzle)

$=P[(E_1 \text{ and } \bar{E}_2) \text{ or } (E_2 \text{ and } \bar{E}_1)]$

$=P(E_1\cap\bar{E}_2)+P(E_2\cap\bar{E}_1)$

$=P(E_1)\times P(\bar{E}_2)+P(E_2)\times P(\bar{E}_1)$

$=\frac{1}{4}\times\frac{4}{5}+\frac{1}{5}\times\frac{3}{4} \qquad [\because P(\bar{E}_1)=1-P(E_1)]$

$=\frac{4}{20}+\frac{3}{20}=\frac{7}{20}$

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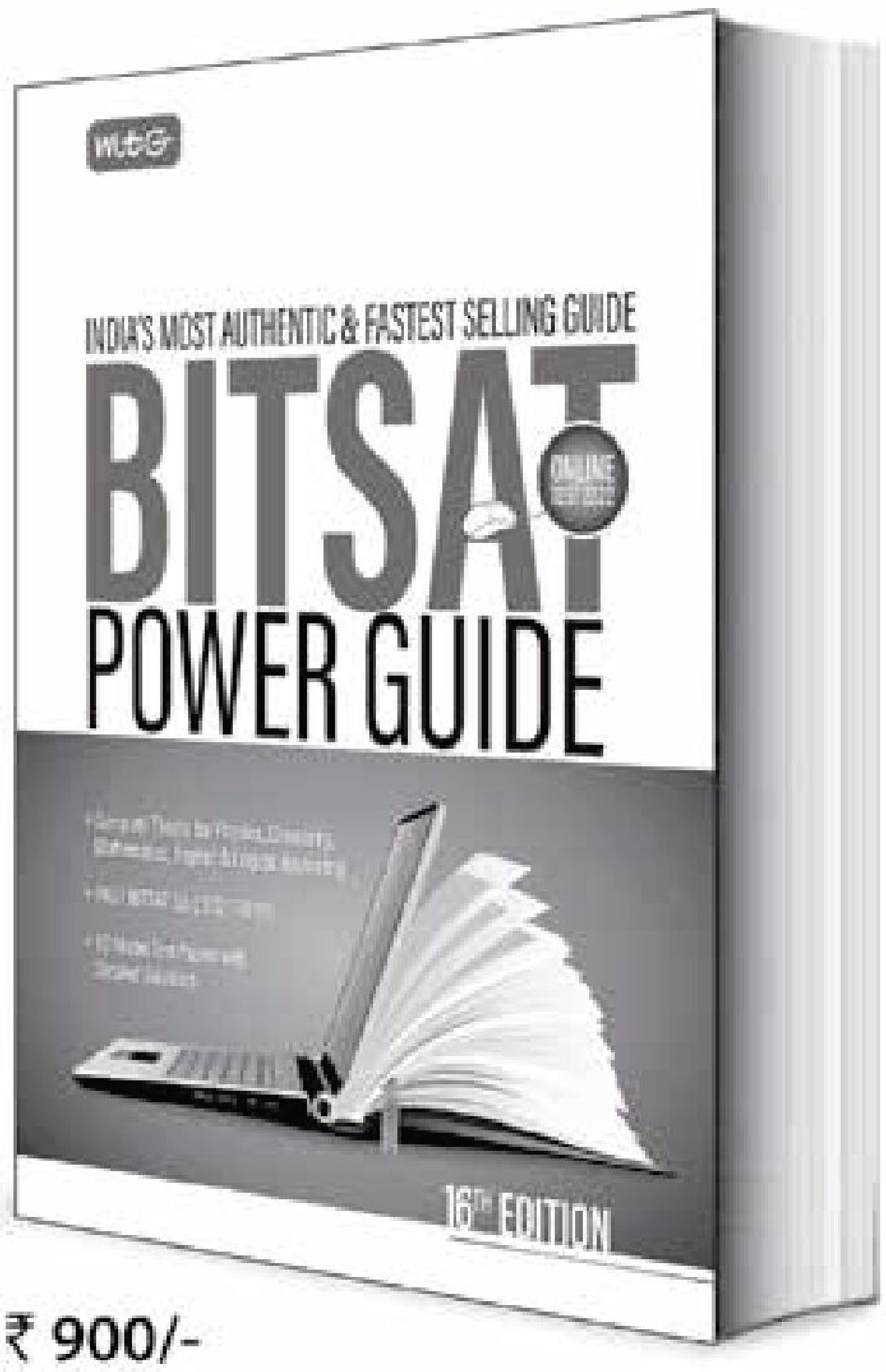

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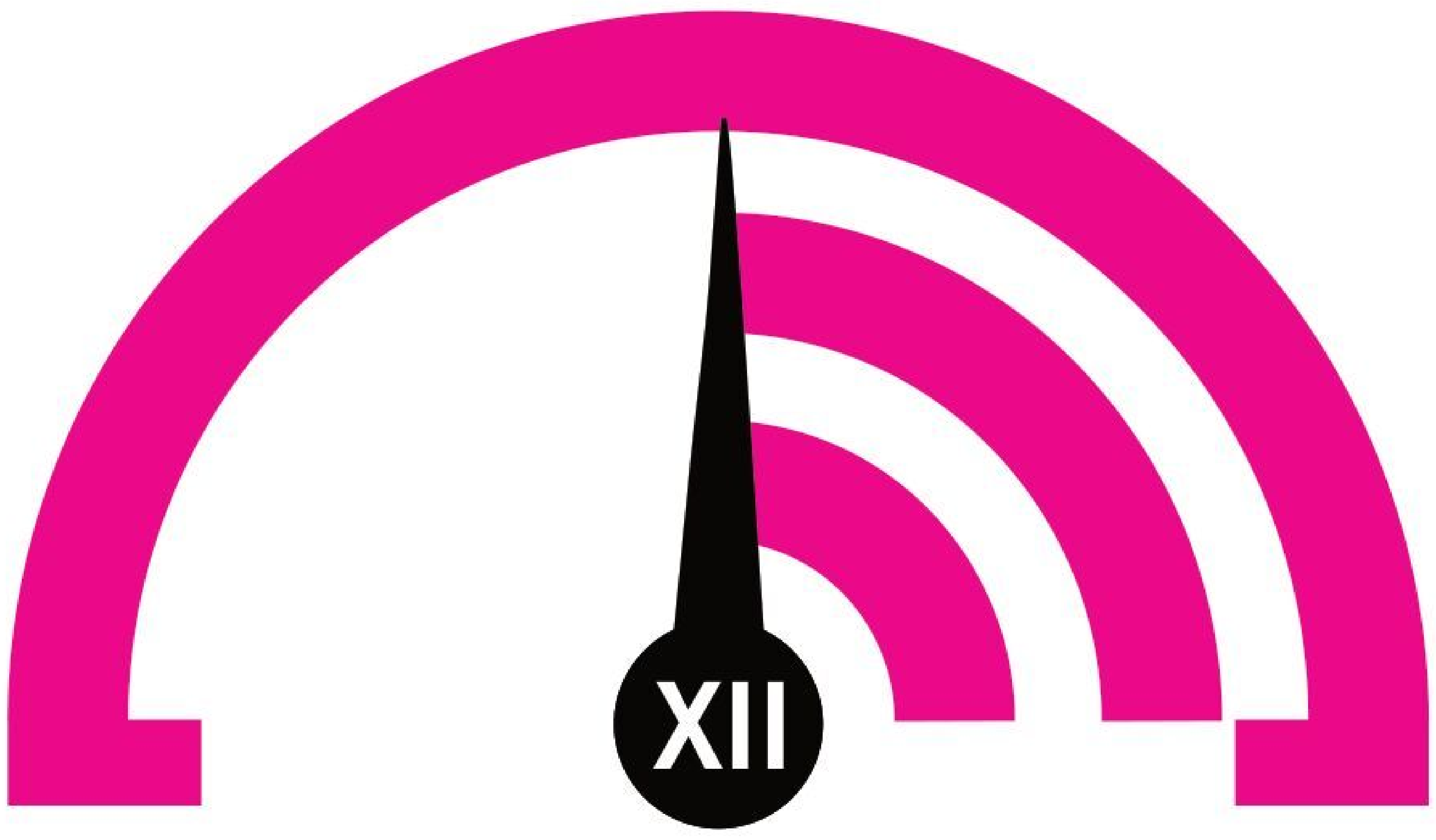

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. $\int_1^e (2x^2+1)(x^2+\ln x)^5\,dx+\int_1^e (x^2+\ln x)^6 dx$

(a) $(e^2+1)^6-1$ (b) $e(e^2+1)^6-1$
(c) $e^2(e^2+1)^5-1$ (d) $e(e^2+1)^5-1$

2. The minimum value of $z = 3x + 5y$ subject to $3x + y \geq 15$, $x + 2y \geq 10$, $x, y \geq 0$ is

(a) 15 (b) 25 (c) 27 (d) 30

3. Let $f(x) = \int_0^x e^x \sin^{-1}(x-1)\ \ln x\, dx$ $(x < 0)$, then

(a) $f(x)$ has one local minima
(b) $f(x)$ is an increasing function
(c) $f(x)$ is a decreasing function
(d) none of the above

4. Let $f(x) = \begin{cases} x+1, & x<0 \\ |x-1|, & x\geq 0 \end{cases}$ and $g(x) = \begin{cases} x+1, & x<0 \\ (x-1)^2, & x\geq 0 \end{cases}$

Then the number of points at which the function $g(f(x))$ is not differentiable is

(a) 0 (b) 1 (c) 2 (d) 3

5. $\int\limits_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x}dx =$

(a) ln 2/20 (b) ln 2/10 (c) ln 3/20 (d) ln 3/10

6. The range of values of p for which the equation $\sin\cos^{-1}(\cos(\tan^{-1}x)) = p$ has a solution is

(a) $\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$ (b) $[0, 1)$
(c) $\left[\frac{1}{\sqrt{2}},1\right)$ (d) $(-1, 1)$

One or More than One Option(s) Correct Type

7. Let E and F be two independent events. The probability that exactly one of them occurs is 11/25 and the probability of none of them occurring is 2/25. If $P(T)$ denotes the probability of occurrence of the event T, then

(a) $P(E)=\frac{4}{5}, P(F)=\frac{3}{5}$ (b) $P(E)=\frac{1}{5}, P(F)=\frac{2}{5}$
(c) $P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$ (d) $P(E)=\frac{3}{5}, P(F)=\frac{4}{5}$

8. A differentiable function satisfies

$$f(x)=\int\limits_0^x (f(t)\cos t-\cos(t-x))dt.$$

Which of the following hold(s) good?

(a) $f(x)$ has a minimum value $1 - e$
(b) $f(x)$ has a maximum value $1 - e^{-1}$
(c) $f''\left(\frac{\pi}{2}\right)=e$ (d) $f'(0) = 1$

9. Let $f''(x) > 0\ \forall\ x \in R$ and $g(x) = 2f\left(\frac{x^2}{2}\right)+f(6-x^2)$. Then

(a) g attains maximum at $x = \pm 1$
(b) g attains minimum at $x = \pm 1$
(c) g attains maximum at $x = 0$
(d) g attains minimum at $x = \pm 2$

10. The equations of two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at an angle 60° is

(a) $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ (b) $\frac{x}{1}=\frac{y}{-1}=\frac{z}{2}$
(c) $\frac{x}{2}=\frac{y}{1}=\frac{z}{-1}$ (d) $\frac{x}{2}=\frac{y}{-1}=\frac{z}{1}$

11. Given a real valued function f such that

$$f(x)=\begin{cases} \dfrac{\tan^2 x}{(x^2-[x]^2)}, & \text{for} \quad x>0 \\ 1, & \text{for} \quad x=0 \\ \sqrt{\{x\}\cot\{x\}}, & \text{for} \quad x<0 \end{cases}$$

Where $[x]$ is the integral part and $\{x\}$ is the fractional part of x, then

(a) $\lim\limits_{x\to 0} f(x)=1$ (b) $\lim\limits_{x\to 0^-} f(x)=\sqrt{\cot 1}$

(c) $\cot^{-1}\left(\lim\limits_{x\to 0^-} f(x)\right)^2=1$

(d) f is continuous at $x=0$

12. If $a^2+b^2-ab-a-b+1\le 0$, $a, b, \in R$ and $f(x)=(1+\sec 2x)(1+\sec 2^2x)\ldots(1+\sec 2^n x)$, then $f\left(\dfrac{\pi}{2^n-1}\right)$ is

(a) $a-b$ (b) $a+b$ (c) ab (d) $\dfrac{a^2+b^2}{2}$

Comprehension Type

Paragraph for Q. No. 13 to 15

Consider the system of equations

$2x+ay+6z=8$, $x+2y+bz=5$ and $x+y+3z=4$, then

13. This system of equations will have infinite solutions if

(a) $b=3$ (b) $b=2$
(c) $a=3$ (d) $a=2$

14. The given system of equations will have no solution if

(a) $a=2, b\ne 3$ (b) $a\ne 2, b\ne 3$
(c) $a\ne 2, b=3$ (d) $a=2, b=3$

15. The given system of equations will have a unique solution if

(a) $a\ne 2, b\ne 3$ (b) $a\ne 2, b=3$
(c) $a=2, b=3$ (d) $a=2, b\ne 3$

Matrix Match Type

16. Match the following :

	Column-I		Column-II
P.	Area between $y=\lvert\cos^{-1}(\sin x)\rvert-\lvert\sin^{-1}(\cos x)\rvert$ and x-axis, where $x\in\left[\dfrac{3\pi}{2},2\pi\right]$ is $\dfrac{k\pi^2}{4}$, then k is equal to	1.	6
Q.	The difference between the radii of the smallest and the largest circles which have their centres on the circumference of the circle $x^2+y^2+2x+4y-4=0$ and passes through the point $(2, 2)$ is	2.	1
R.	If $f(0)=1$, $f(2)=3$ and $f'(2)=5$, then $\int_0^1 xf''(2x)\,dx$ is equal to	3.	-2
S.	If $f(x)=x^3+4x^2+6x$, then $f^{-1}(-4)$ is	4.	2

	P	Q	R	S
(a)	2	1	4	3
(b)	1	2	3	4
(c)	4	3	1	2
(d)	3	2	1	4

Numerical Answer Type

17. Let $\vec{b}=4\hat{i}+3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy-plane. The dot product of the vector in the same plane having projection 1 and 2 along $\vec{b}$ and $\vec{c}$ with vector $3\hat{i}+\hat{j}$ is

18. Let $\displaystyle\int\frac{(4+3x)\,dx}{(x^3-2x-4)}=\ln\left|\frac{x-k}{\sqrt{x^2+kx+l}}\right|+C$, the k^2+l^2 is equal to

19. The order and degree of the differential equation satisfying $\sqrt{1-x^4}+\sqrt{1-y^4}=a(x^2-y^2)$ are P and Q respectively, then $P+Q$ is (a is arbitrary constant)

20. If $\dfrac{|x-1|}{|x|-1}\,|\ln x|=kx$, has one solution, then number of non negative values of k will be

◆

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No. of questions correct ……
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90-75%	**GOOD WORK !**	You can score good in the final exam.
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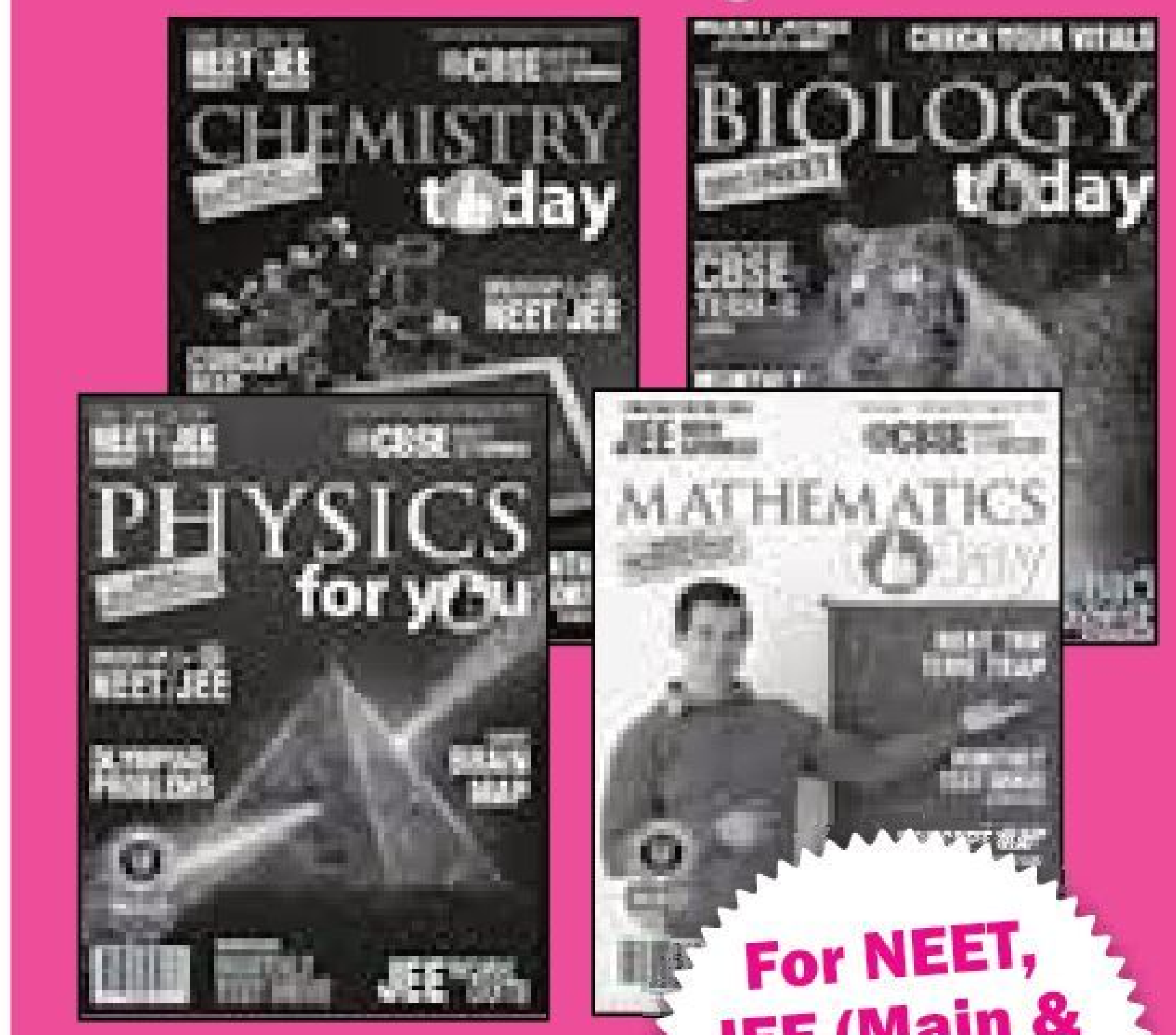

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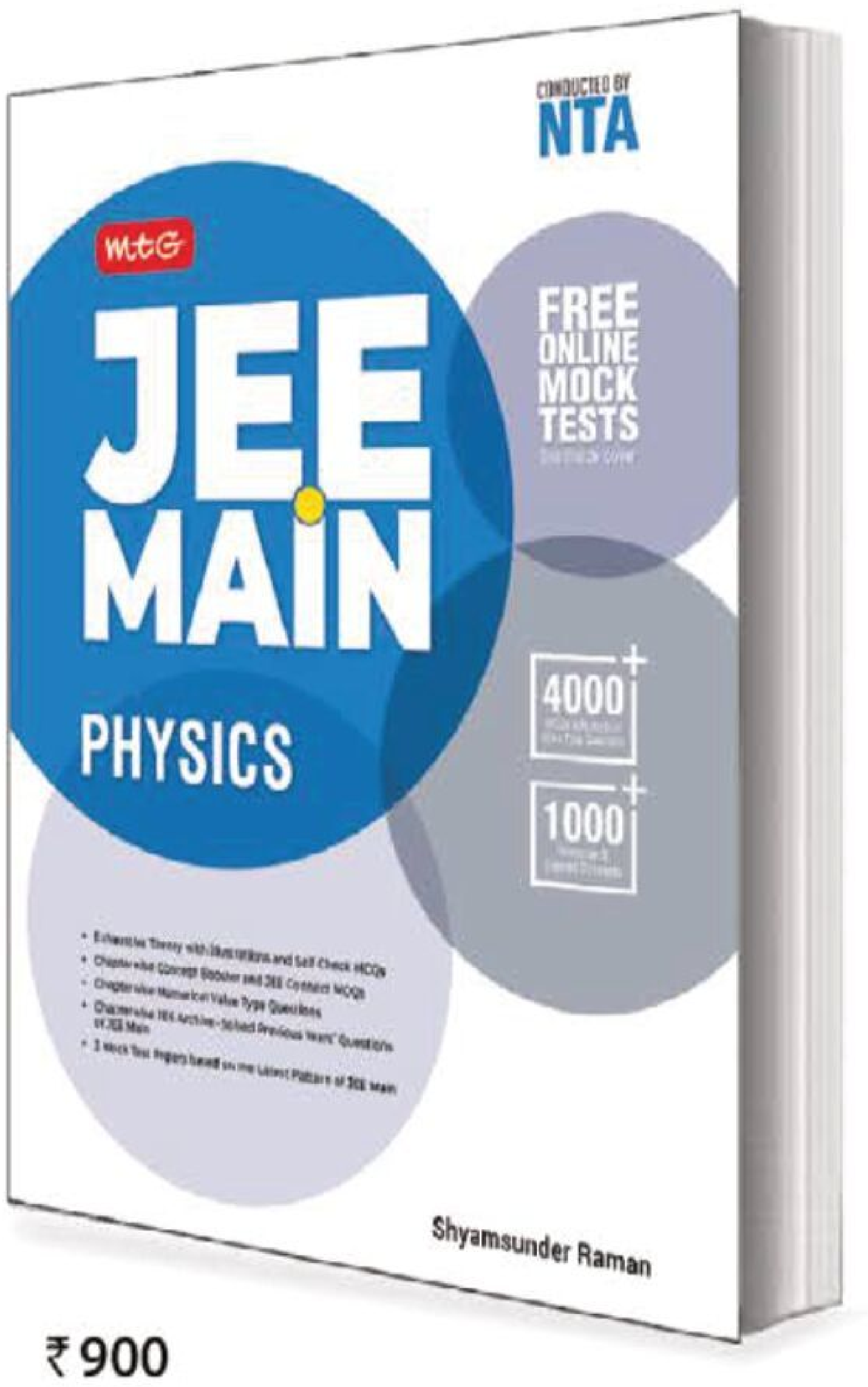

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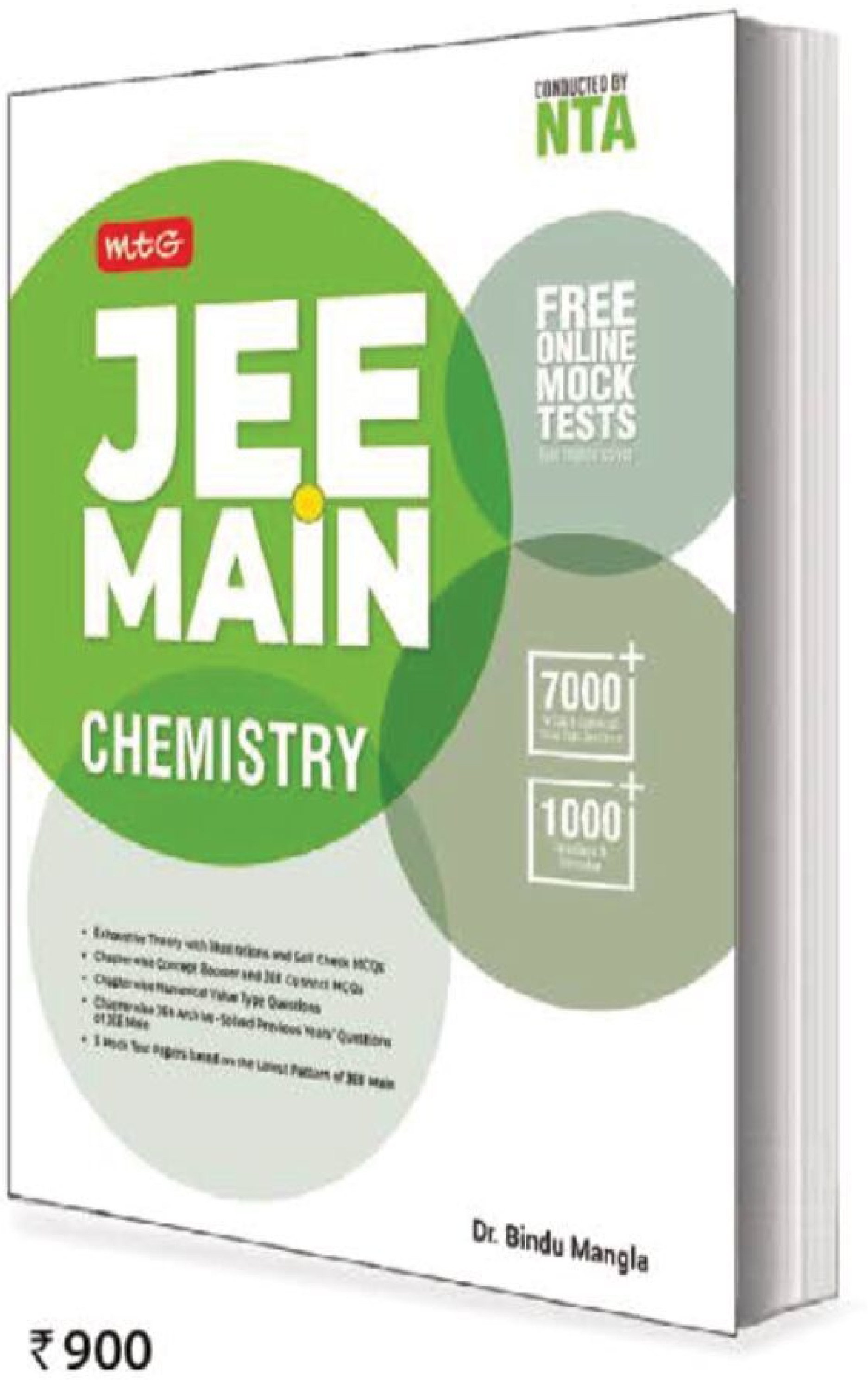

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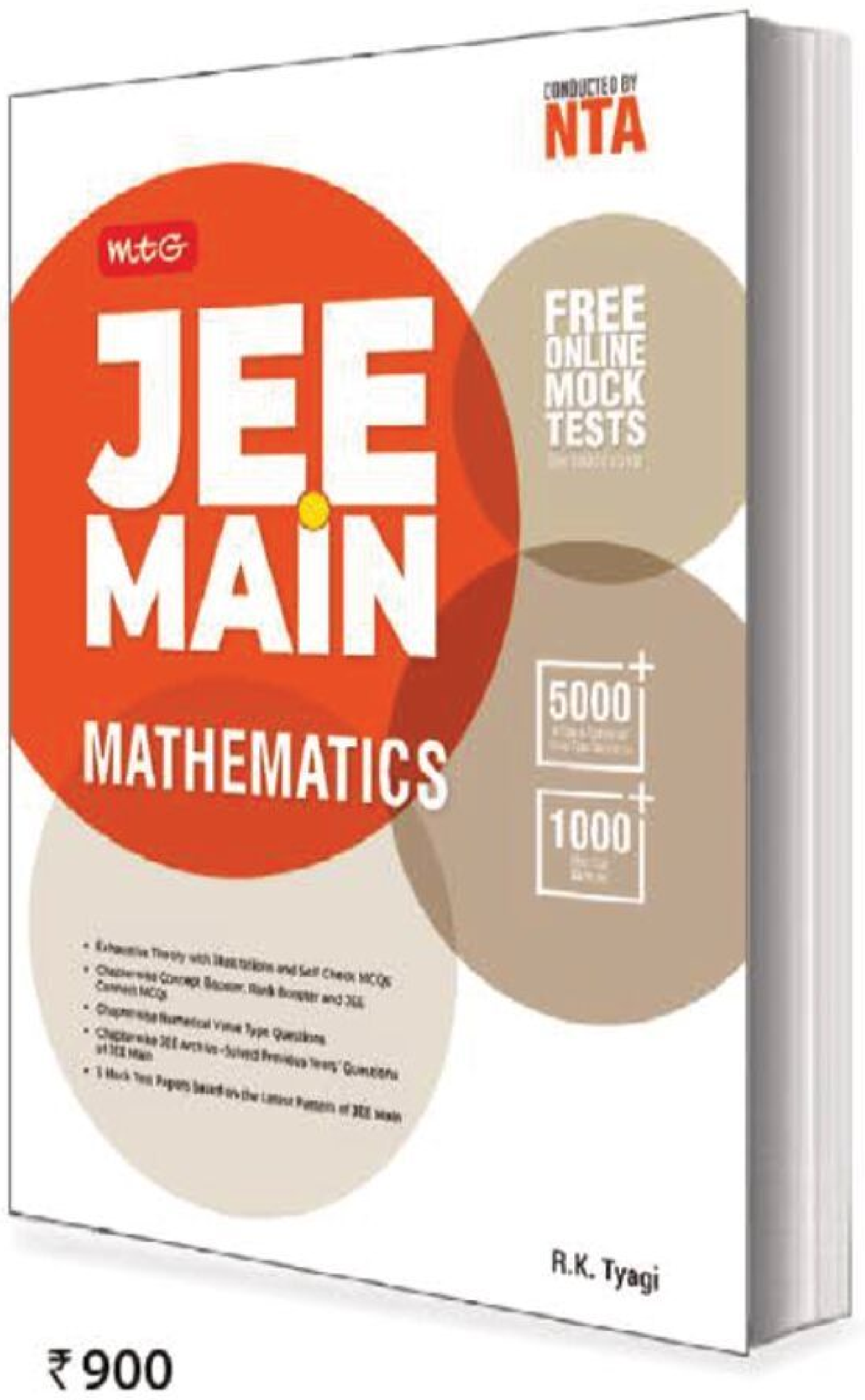

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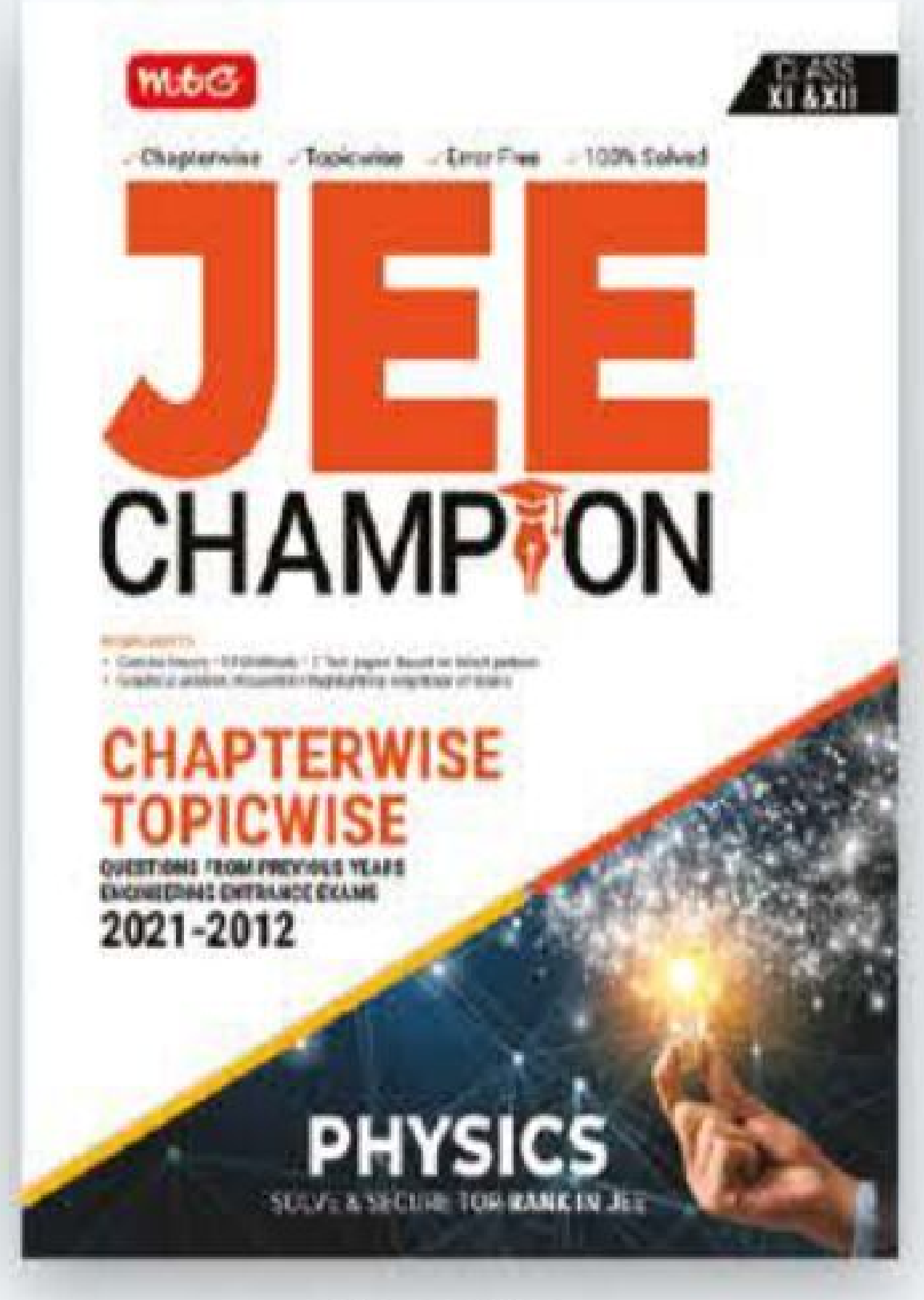
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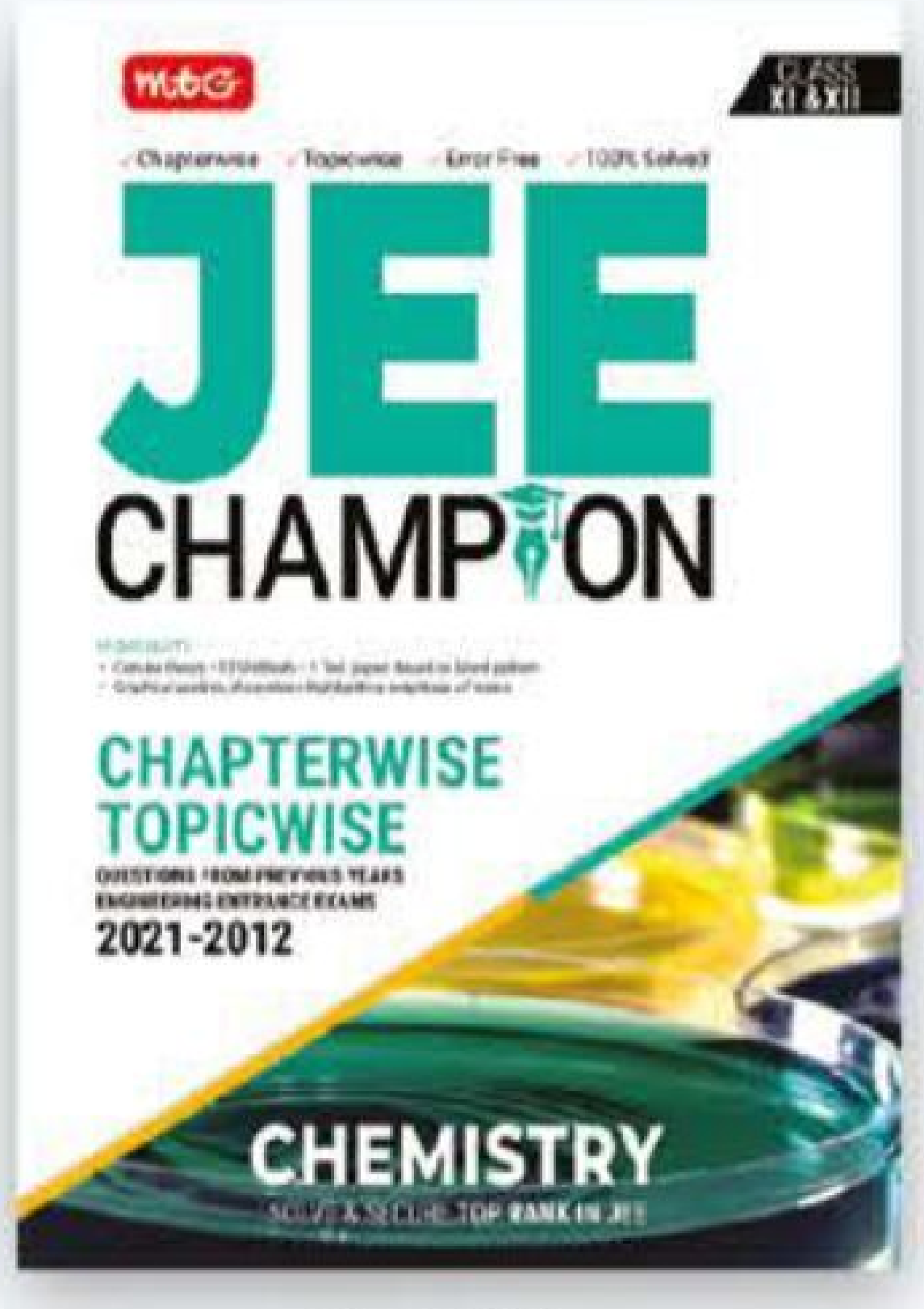
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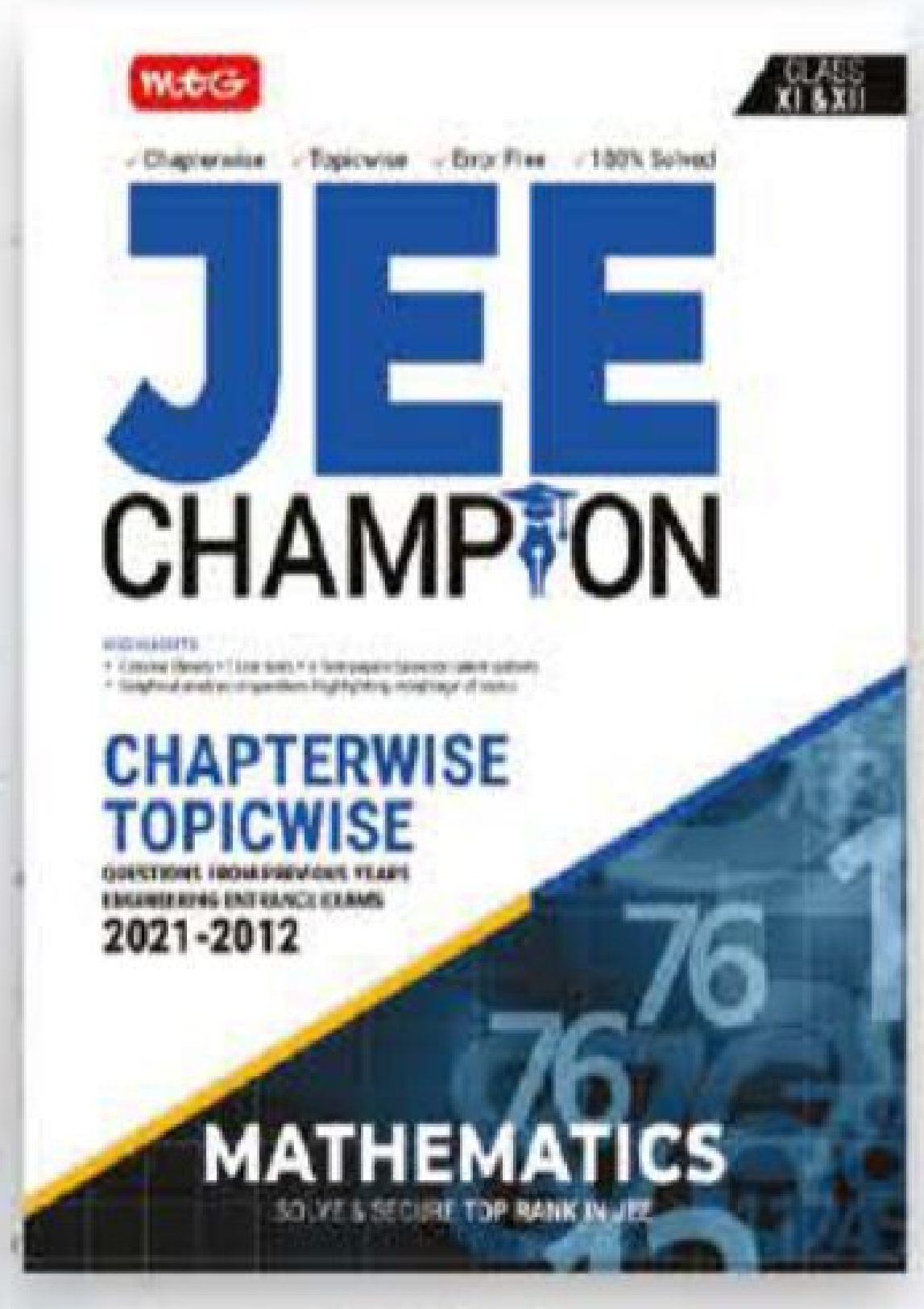
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